

# Portfolio analysis and geographical allocation of renewable sources: A stochastic approach

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## ABSTRACT

We take inspiration from the Modern Portfolio Theory introduced by Markowitz to propose a simplified strategy for the portfolio management of renewable energy sources based on Gaussian fluctuations with tunable correlations. By analyzing the impact of production fluctuations, we show how – depending on the sources' temporal correlation patterns – a careful geographical allocation of different types of renewal energy sources can reduce both the energy needs for balancing the power system and its uncertainty. The proposed strategy can be easily integrated in a decision support system for the planning of renewable energy sources. Therefore, providing policy/decision makers with an additional tool. We test our strategy on a set of case studies including a real-case based on literature data for solar and wind sources, and discuss how to extend the computation to non-Gaussian sources. The paper shows that in the Markowitz framework an efficient trade-off between production and fluctuations can be easily achieved, and that such framework also leads to important considerations on energy security. In perspective, analysis of time series together with such enriched frameworks would allow for the analysis of multiple realistic renewable generation scenarios helping decisions on the optimal size and spatial allocation of future energy storage facilities.

## 1. Introduction

Taking into account the recent environmental disasters, the increase in pollutant emissions (especially in large urban areas like megacities Kennedy et al., 2015) and the increasing risk of climate change in coastal areas suggests that the exploitation of planetary resources as infinite source disregarding of the environmental impact of human activities should no longer be considered as a driving force of world economies. Considering this scenario, Green and Sustainable Economy is gaining importance not only as simple branch of ecological economics, but represent a concrete economic development model. For example, in the last thirty years an energy policy based on sustainable development has been implemented by the European Union as a foundational act, in accordance with the post-Kyoto protocol and the formalization of the package “Climate-Energy 20-20-20” (in force since June 2009 and valid from January 2013) which has three main objectives to be achieved by 2020: (1) 20% emissions reduction compared to 1990 levels; (2) reaching the renewable share of 20% compared to

the gross final consumption; (3) improving energy end-use efficiency of 20% (Cucchiella et al., 2017) The implementation of such incentive policies favoured in the last years the spread of renewable energy sources (mainly wind and photo voltaic), considered almost inexhaustible while ensuring a drastically reduced environmental impact when compared to traditional sources such as fossil fuels.

With regards to the impact of renewable energy sources on the power grid, production from intermittent renewable sources brings with it challenges in terms of matching demand and supply on a real time basis (Brouwer et al., 2014). The limited programmability of solar and wind generation capacity has increased the amount of less efficient and more polluting spinning reserve generation needed to be called upon at any moment by the balancing market (Ortega-Vazquez and Kirschen, 2009), partly offsetting the environmental and economic advantage of such renewable sources (Facchini, 2017). Although battery storage is often cited as a definite solution to this problem, it is still expensive and hardly scalable in a short time frame (Weitemeyer et al., 2015), and not always is a suitable and viable solution under the

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economic point of view (Korjani et al., 2017). Although innovative solutions like using a carefully planned electric car mobility for buffering energy production fluctuations have been proposed and analysed (Mureddu et al., 2018), we are still far away from an agreement of a common storage strategy. Therefore, an albeit more limited but sensible approach would be to investigate whether an optimal spatio-temporal allocation of renewable energy production sites could reduce fluctuations and thus the volume of electricity produced and sold on the balancing market (Mureddu et al., 2015). Limiting the fluctuations would not only reduce the size of balancing markets and of greenhouse gas emissions, but would also be beneficial in reducing the impact on the grid and on the storage systems, especially in terms of aging cycles and optimal charge management of storage systems (Xu et al., 2016; Korjani et al., 2017).

With regards to the optimal management of renewable sources, in recent years Portfolio Theories have found a set of interesting applications. Portfolio Theory is nowadays a widely accepted method for the selection of efficient energy generation portfolios (Bhattacharya and Kojima, 2012; Kruyt et al., 2009) with the aim to maximize social aspects (Awerbuch and Berger, 2003) or financial returns (Roques et al., 2008). The method generally makes use of numerical methods to determine the most efficient mix of generation sources, also applying an empirical/analytical approach (Sunderkötter and Weber, 2012). Some authors also use the Sharpe Index (Janczura, 2010) to measure the performance of different energy portfolios composed of oil, coal, natural gas, nuclear energy and renewable sources. Awerbuch and Berger (2003) provided one of the first evaluations of the application of portfolio theory for development of efficient portfolios with diversification of sources applied to the original 15 EU nations. With regards to EU countries, Garcia et al. developed a model for the management of different sources at company level (Garcia et al., 2017), while Cucchiella and colleagues applied the portfolio concept to the Italian electricity market (Cucchiella et al., 2017), showing that Portfolio theory can help in planning better and greener renewable sources production schemes. For a review on applications of Portfolio theory to energy systems the reader is referred to DeLlano-Paz et al. (2017).

Following the above mentioned works, in this paper we investigate a new aspect related to the geographical allocation of renewable sources based on their temporal correlation characteristics; such a model allows to consider the interplay among expected energy production and RES production fluctuations, since the variability in energy production must be balanced by traditional power sources, hence not only increasing the production of pollutants but also customers' bills, since electric power on the balancing market has much higher costs. To this aim we introduce a simplified model to investigate the power output of renewable energy sources by means of Gaussian fluctuations with tunable correlations, analyzing the impact of production fluctuations via Modern Portfolio Theory (MPT) analysis introduced by Markowitz (1959). Our aim is to provide an additional criterion to be integrated into planning/optimization tools for renewable energy sources placement that can be used also for greener energy policies. In this paper we show how a careful geographical allocation, based on the temporal correlation patterns of the renewal energy sources, can reduce both the amount of energy needed for balancing the power system and its uncertainty. We then discuss an extended framework that allows for the optimization of real non-Gaussian portfolio of renewable energy sources (both wind and solar) showing that an efficient configuration of renewable power plants is easily achievable through MPT. In perspective, analysis of time series (Facchini et al., in press) together with such enriched frameworks would allow for the analysis of multiple realistic renewable generation scenarios helping decisions on the optimal size and spatial allocation of future energy storage facilities.

This paper is organized as follows: In Section 2 we introduce the stochastic model and discuss its impact on the assessment of energy security and congestions; moreover, we consider its extension to the case of non-Gaussian time series. In Section 3 we present a case study

based on synthetic energy sources showing that the beneficial effects of negative correlations on portfolio allocation; we then present a case study based on real data from Italian renewable sources, showing how even a simple balancing of wind and solar power could reduce the size of the fluctuations. Finally, we state our conclusions and future directions in Section 4.

## 2. Model description

In this section, we follow the ideas of financial portfolio for optimal investments developed by Markowitz (1959). Portfolio theory is based on the financial principle of asset diversification. According to this approach the diversification of financial assets can lower the overall risk compared to the risk of the individual assets. Finance theory divides total risk into two different components: (1) unsystematic risk, principally affecting the prices of an asset (this risk can be reduced by means of diversification) and (2) systematic risk, affecting the prices of all assets. Systematic risk cannot be diversified and is connected to a common risk related to all securities (market). A portfolio is efficient when the unsystematic risk is removed through the diversification and, accordingly, the market portfolio risk (standard deviation) equals the systematic risk. Furthermore, each financial asset is composed by three elements: (1) the expected return of the asset, (2) the risk of the asset (i.e. the standard deviation of the expected returns due to market fluctuations, also called *volatility* of the asset) and (3) the correlation between the assets composing the portfolio. The correlation between assets measures how two assets behave in relation to each other. For example, a perfect positive correlation (+ 1) implies that the two assets move in the same direction. Perfect negative correlation (− 1) means that the two assets move in the opposite direction. If the correlation is 0 the two assets fluctuate randomly. The risk is minimized according to a specific return as objective and an optimal portfolio is formed by assets negatively correlated. Investors do not necessarily aim to maximize return, but also take into account the risk associated with investments and the realization of portfolios that decrease the risk by maintaining the value of return.

We introduce an optimal portfolio for renewable energy sources by considering the need to allocate a set of  $N$  renewable electric power plants on  $i = 1 \dots N$  sites, and we call  $W_i$  the maximum possible power generation size on the  $i$ -th site. To describe the sites' energy production, we will assume that their unit production is described by a random variable  $p_i$ , with expected value  $E(p_i) = e_i$  and fluctuations characterized by their standard deviation  $\sigma_i$ , where  $\sigma_i^2 = E[(p_i - e_i)^2]$ ; notice that in the language of Portfolio Theories  $\sigma_i$  is also called the *risk* associated with the  $i$ -th source. Notice that a single source can refer also to a set of geographically nearby micro-generators, like the ones that can be planned in cities to enhance self-production and try to match locally demand and generation in the form of microgrids (Halu et al., 2016), a strategy that is considered promising to achieve the goal of net negative electric cities (Kennedy et al., 2018, 2017; Stewart et al., 2018).

In our case, since we are considering electric generation, the produced power must exactly match the demand and that any fluctuation needs to be balanced by the grid and by the balancing market. Hence, also within this framework, variance is strictly related to the risk of diminished revenues or even economic losses because of fees paid in the balancing market.

We will indicate the energy output of a plant as  $p_i w_i$ , where  $w_i$  is the size of the  $i$ -th plant, while the set  $\mathcal{W}$  of all the possible allocations (i.e. the set of all the generation possibilities of the  $N$  power plants) is constrained by the  $N$  inequalities:

$$\mathcal{W} = \{w_i: 0 \leq w_i \leq W_i\} \quad (1)$$

and hence is a convex set. We will also refer to the vector  $\vec{w} \in \mathcal{W}$  subject to the constraints reported in Eq. (1) as a *energy generating portfolio* or, more simply, a *portfolio*.

Since we are considering renewable plants, their energy outputs will

be correlated because of the fact that, usually, the sites where they are deployed are close under the geographical point of view, therefore subject to similar weather conditions influencing their production and their fluctuations. As a first approximation, we will describe such correlations by their covariance matrix  $\sigma$  whose elements are  $\sigma_{ij} = E[(p_i - e_i)(p_j - e_j)]$ , and  $e_i$  is the expected production from site  $i$ . By definition, the correlation coefficient between variables  $i$  and  $j$  is defined by  $\rho_{ij} \sigma_i \sigma_j = \sigma_{ij}$ . We can thus compute the corresponding covariance matrix  $C$  as:

$$C_{ij} = \sigma_i \rho_{ij} \sigma_j \tag{2}$$

Under such hypothesis, the total production from renewable sources  $P_R$  will be a random variable  $P_R = \sum w_i p_i$  with expected value  $E_R$  and variance  $\sigma_R$

$$\begin{aligned} E_R &= \sum_i w_i e_i \\ \sigma_R^2 &= \sum_i \sum_j w_i w_j \sigma_{ij} = \sum_i \sum_j w_i w_j \rho_{ij} \sigma_i \sigma_j \end{aligned} \tag{3}$$

Hence,  $E_R(\vec{w}) = \vec{w} \cdot \vec{e}$  is a linear functional of the allocation vector  $\vec{w}$ , while  $\sigma_R^2(\vec{w}) = \vec{w} \cdot C \vec{w}$  is a quadratic form of the allocations  $\vec{w}$  in the covariance matrix  $C$ . Following MPT, this is equivalent to consider a Markowitz portfolio with expected return  $E_R$  and volatility  $\sigma_R$ . Accordingly, we consider the expected return  $E_R$  as the production that one can obtain from the  $N$  generators, and the volatility as the power fluctuations that the generators produce. Therefore, a portfolio can be considered optimal either when for a given level of generation it achieves the minimum value of the fluctuations, or when for a given level of risk (i.e. given the size of the fluctuations) it achieves the maximum level of generation.

The set  $\mathcal{P}_R$  of the feasible renewable energy productions  $(E_R, \sigma_R)$  is the image of a compact convex set  $\mathcal{W}$  via linear and quadratic functions; hence, also  $\mathcal{P}_R$  is a compact convex set. Optimal portfolios correspond to those  $\vec{w}$  whose image  $(E_R(\vec{w}), \sigma_R(\vec{w}))$  lies on the upper boundary of  $\mathcal{P}_R$ . In Fig. 1 we report a general example of feasible portfolios. In particular, the set  $\mathcal{P}_R$  of feasible portfolios is located in the internal area, while the *efficient frontier*, i.e. the boundary of  $\mathcal{P}_R$  corresponding to optimal portfolios, corresponds to the upper branch of the curve.

It is worth noticing that at each point in the  $(E_R, \sigma_R)$  plane can correspond more than a portfolio, and that not only the best, but also the worst portfolios are found on the boundaries of  $\mathcal{P}_R$ . The choice of an

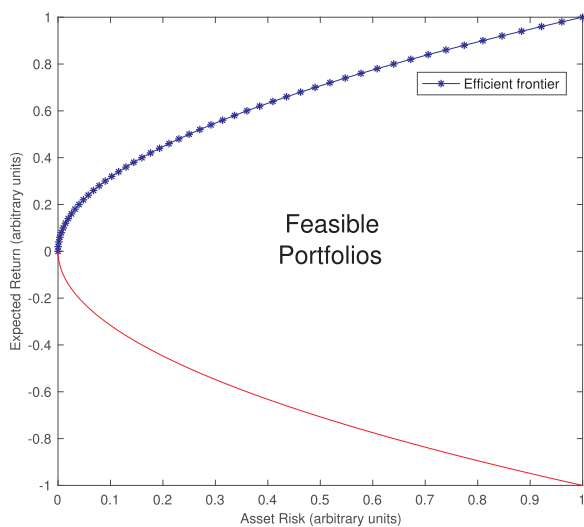


Fig. 1. General example of efficient portfolio boundaries. The frontier of the feasible portfolios corresponds to the case of high production with small uncertainties, and corresponds to the upper part of the parabola.

optimal portfolio depends on the desired balance between the increasing revenues of higher expected energy production and the increasing costs associated with balancing fluctuations in the energy production; since such constraints will fix only a value of  $(E_R, \sigma_R)$  on the efficient frontier, there will be in general freedom to decide which of the portfolios corresponding to such values to choose.

### 2.1. Convexity and investment costs

We now consider the effects of the costs on the allocations. We have already assumed that physical, geographical, and political constraints limit the size of the plants (i.e. of the portfolio) in the convex set  $\mathcal{W}$  defined as the Cartesian product of finite intervals  $0 \leq w_i \leq W_i$  of feasible sizes. On the other hand, the plants allocation will be naturally limited by a finite budget  $B$ , constraining the total investment cost to be  $0 \leq \sum f_i(w_i) \leq B$ , where  $f_i$  is the cost function associated to the  $i$ -th plant. If – as reasonable – the production cost of each plant is a monotonic non-decreasing function, then also the set  $\mathcal{B} = \{w_i: 0 \leq \sum f_i(w_i) \leq B\}$  of possible investments is convex. This also implies that the set of feasible points  $\mathcal{F} = \mathcal{W} \cap \mathcal{B}$  (i.e. the feasible portfolios) is also a convex set. Finally, the attainable productions  $\mathcal{P} = \{(E(\vec{w}), \sigma(\vec{w})) : \vec{w} \in \mathcal{F}\}$  is also a convex set, since it is obtained as the image of a convex set via a positive definite quadratic form; moreover, the images of optimal portfolios will be allocated on the boundaries of  $\mathcal{F}$ . For simplicity, in this paper all the examples will be worked out in the case where the investment cost does not depend neither on the technology applied nor on the geographical location, but will reflect only realistic fluctuations of RESs productions on the Italian ground as modelled in the data-set of Mureddu et al. (2015).

### 2.2. Considerations about energy security

Energy security, i.e. the uninterrupted availability of access to energy sources, is potentially hampered by fluctuations in energy production; in this regard, it is well recognized that the introduction of RESs must be carefully planned (Hammons, 2008; Kunz, 2013), especially since RESs have priority dispatch. In general, fluctuating sources can change the flows in a power grid in a way that was not planned when the infrastructure was deployed, hence leading to unexpected energy congestions; as an example, theoretical investigations indicate that enhancing the presence of fluctuating energy sources could even lead to an increased probability in blackouts (Pahwa et al., 2014).

To quantify the risk of energy congestions, we now consider the case where a constant energy demand  $D$  needs to be satisfied both by non-intermittent (i.e. “traditional”) energy plants – capable of a constant energy production  $E_0$  – and by renewable sources with a fluctuating production  $P$ . Since consumption and production must be always balanced, the stochastic variable  $\Delta = E_0 + P - D$  represents the size of the balancing market. We remark that the balancing of  $\Delta$  is implemented through traditional energy sources with a stronger carbon footprint.<sup>1</sup>

As an example, supposing that:

- (1) The average renewable production is able to balance the demand, i.e.  $E_0 + E_R = D$ ;
- (2) The balancing market is able to absorb all the fluctuations up to a size  $\delta$ ;

then the probability  $\alpha = Prob[-\delta \leq \Delta \leq \delta]$  that the balancing market is able to absorb fluctuations is a decreasing function of the renewables’ production variance  $\sigma_R$ .

In this special case, if renewable plants’ productions can be

<sup>1</sup> A carbon footprint is defined as the total emissions caused by an individual, event, organisation, or product, expressed as carbon dioxide equivalent.

described with stochastic Gaussian variables, and  $\alpha$  can be calculated in closed form as  $\alpha = 1 - 2 \operatorname{erf}(\delta/\sigma_R)$ . However, this would also provide a reliable estimate in the case of large number of non-Gaussian production  $p_i$  with finite average and variance (as is usually in the case of high penetration of distributed RES).

Hence, the portfolios  $\vec{w}$  that maximize security, i.e. the presence of manageable fluctuations, are:

$$\min_{\vec{w} \in \mathcal{F}} \operatorname{erf}(\delta/\sigma_R(\vec{w})) \tag{4}$$

and correspond exactly to the ones that minimize  $\sigma_R(\vec{w})$ . This result will be helpful in the next section, where we will consider the case of non-Gaussian sources both from wind and solar data.

### 2.3. Time dependent fluctuations

Since renewable sources are linked to the weather, the average values of the production can be strongly time dependent. This is particularly true for solar plants, where production is absent during the night and shows a peak around noon; moreover production will be stronger in the summer and lower in the winter. Also in the case of winds there will be seasonal components and possibly daily components if breeze regimes are present. On the same footing, also the size of the fluctuations could be time dependent. For example, we could divide the time in discrete intervals labelled by the variable  $t$ ; then we can indicate as  $e_t^f$  the average production and  $\sigma_t^f$  its standard deviation at time  $t$ . Thus, also optimal portfolios would depend on the time  $t$ . Since we are looking for a static allocation of our energy sources, we must reduce to the case of a single portfolio. A possible approach would be to transform energy production in money; as an example, if  $r^t(\epsilon)$  is the revenue for producing  $\epsilon$  units of energy at time  $t$  and  $l^t(\epsilon)$  as fee to pay if the actual production differs by  $\epsilon$  from the expected value, one can go back to the original MPT formulation with  $e_i = \sum_t r^t(e_i^f)$  and  $\sigma_i = \sum_t l^t(\sigma_i^f)$  as long as  $l^t(\epsilon)$  is a linear function. If this is not the case, it is necessary to resort to numerical intensive techniques like Monte Carlo methods or genetic algorithms. We notice that, if complete time series are at hand – either from stochastic models or from real data-sets based on the past – all the quantities can be estimated through either Monte Carlo sampling or by numerical simulations. As an example, using time series of the weather can help assess the impact of cyclic components, while using predictive climate models could help investigate possible forthcoming scenarios.

## 3. Results

In this section we show how to compute the optimal portfolio in different case studies. We start by considering the simple case of Gaussian sources providing a closed-form solution in the case of  $N = 2$ , and numerically extending it to the case of more Gaussian sources. As a final application example, we use real time series from wind and solar generation to show how the efficient frontier can be computed in the case of non-Gaussian sources.

### 3.1. Gaussian renewable power plants

To have an intuition of the interplay among expected energy produced and the risk of fluctuations, we first consider the simple case of two renewable power plants; in this case, correlations are described by a single number  $-1 \leq \rho \leq 1$ , and we assume a limited amount of resources to be distributed among the two plants subject to linear constraints  $w_1 + w_2 = W$ , i.e.  $w_1 = w$ ,  $w_2 = W - w$ . Therefore, Eq. (3) becomes:

$$\begin{aligned} E_R &= w_1 e_1 + w_2 e_2 \\ \sigma_R^2 &= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_1 \sigma_2 \rho \end{aligned} \tag{5}$$

Since  $\partial E_R / \partial \rho = 0$ , the expected value of the produced power is not

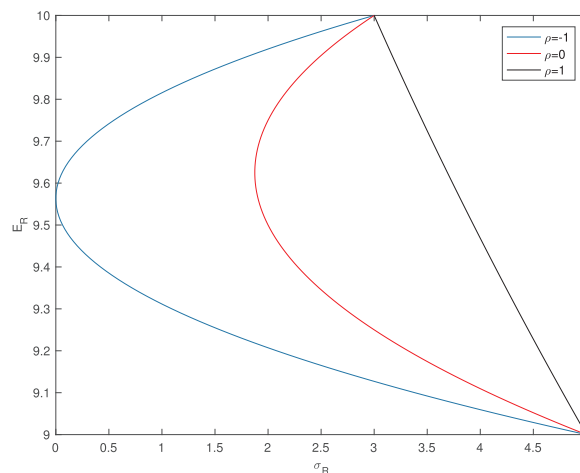


Fig. 2. Values of  $E_R$  and  $\sigma_R$  in the case of two-resources allocation for different correlation values. In the ideal hypothesis of totally anticorrelated sources When sources are totally anti-correlated ( $\rho = -1$ ), it is possible to attain the minimum value for the risk; in general, decreasing the correlation at fixed output  $E_R$  decreases the risk  $\sigma_R$ .

influenced by the fluctuations, while since  $\partial \sigma_R^2 / \partial \rho \geq 0$ , for a given resource allocation, the variance decreases with the correlations and the minimum possible value is attained when the two sources are maximally anti-correlated (i.e.  $\rho = -1$ ).

We show in Fig. 2 a theoretical example of feasible portfolios in the case of two variables for different values of  $\rho$ . For simplicity, we have chosen to vary the allocation  $w_1$  in the interval  $[0, 1]$  with  $w_2 = 1 - w_1$ . According to this choice, there is a one-to one correspondence between  $(E_R, \sigma_R)$  and the two variables  $w_1$  and  $\rho$ ; with such choice,  $E_R$  assumes the two values  $e_1, e_2$  for the extreme allocations  $w_1 = 1, w_2 = 0$  and  $w_1 = 0, w_2 = 1$ . Moreover, we consider the case in which the second plant has a lower expected production ( $e_2 < e_1$ ) but both plants are characterized by the same variance. In this case, since  $\sigma_1 = \sigma_2$ , it is possible to minimize the risk ( $\sigma_R = 0$ ) when the productions of the two plants are totally anti-correlated ( $\rho = -1$ ).

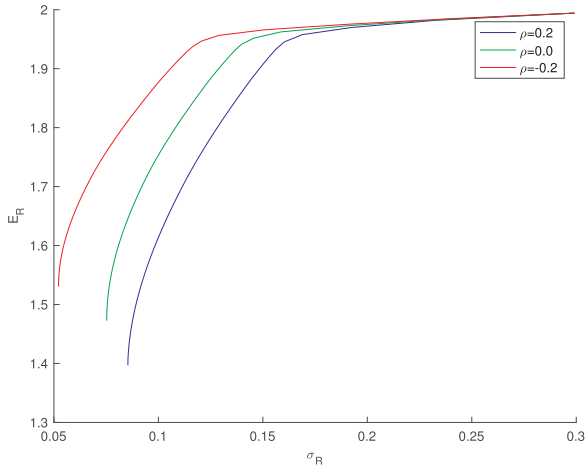
We remark that the optimal portfolio to be chosen strongly depends on political and economic factors. In fact, if there is no penalty for producing fluctuations, the best option for an investor is to take the portfolio with the maximum average production. On the other hand, the best portfolio for the environment is the one with the lowest fluctuations, unfortunately corresponding to a lower power production. In the first case, the owner transfers the extra cost of energy balancing (both in terms of pollution and tariffs) to the community; in the second case the owner is forced to a lower return in his investments. All the other points on the efficient boundary represent possible compromise among the investor and the community that can be reached according to the policy implemented.

### 3.2. Several Gaussian renewable power plants

Extending the model to  $N > 2$  Gaussian sources, we first notice that, while for two variables a maximal anti-correlation  $\rho = -1$  can be attained, this is not generally possible in the case of more variables at the same time, i.e. it is not possible to attain  $\forall i, j \rho_{ij} = -1$ ; mathematically, one has to check that the spectrum of the covariance matrix  $C$  is non negative.<sup>2</sup> We will now consider a synthetic case of  $N_s$  solar plants with average production  $e_i^s$  and risk  $\sigma_i^s$  and  $N_w$  wind-power stations with average production  $e_i^w$  and risk  $\sigma_i^w$ . The total output of a portfolio  $\vec{w}$  is thus  $E_R = \vec{w} \cdot \vec{e}$  where  $\vec{e} = \begin{pmatrix} e^s \\ e^w \end{pmatrix}$ . By indicating with  $\rho_{ij}^{ss}, \rho_{ij}^{ww}$  the mutual

<sup>2</sup> The spectrum of a matrix is defined as the set of its eigenvalues  $\lambda_1, \lambda_2, \dots, \lambda_n$ .





**Fig. 3.** Efficient portfolio boundaries in the case of  $N = 5$  Gaussian sources for different values of the correlation coefficients. Notice that for more than two sources it is possible to reach a negligible value of the risk even if  $\rho > -1$ .

correlation among solar plants and wind stations and with  $\rho_{ij}^{sw}$  the cross correlations among solar and wind production, we can define the reduced covariance matrices  $C_{ij}^{ww} = \sigma_i^w \rho_{ij}^{ww} \sigma_j^w$ ,  $C_{ij}^{ss} = \sigma_i^s \rho_{ij}^{ss} \sigma_j^s$  and  $C_{ij}^{ws} = \sigma_i^w \rho_{ij}^{ws} \sigma_j^s$ . Thus, for a portfolio  $\vec{w} = \begin{pmatrix} \vec{w}^s \\ \vec{w}^w \end{pmatrix}$  its risk is  $\sigma_R = \vec{w} \cdot (C\vec{w})$  with covariance matrix

$$C = \begin{bmatrix} C^{ss} & C^{sw} \\ C^{ws} & C^{ww} \end{bmatrix} \quad (6)$$

For the sake of simplicity, we will consider the simplified case where only (anti)correlation among wind and solar are considered:  $\rho^{ww} = 0$ ,  $\rho^{ss} = 0$  and  $\rho_{ij}^{ws} = \rho \forall i, j$ ; hence, cross correlations among wind and solar outputs become a parameter and  $C^{ws} = -\rho \vec{\sigma}^w \otimes \vec{\sigma}^s$ . Moreover, we will consider average productions (both for wind and solar) to be equally distributed among 0.5 and 1. Following (Mureddu et al., 2015), we will use as values for the fluctuations  $\vec{\sigma}^s = 0.10 \vec{e}^s$  and  $\vec{\sigma}^w = 0.15 \vec{e}^w$ .

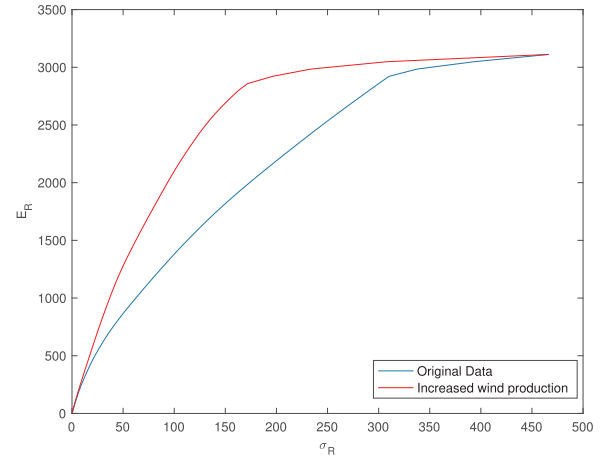
In Fig. 3 we show the results for  $N_s = N_w = 5$  plants at varying  $\rho$ . We notice that it is possible to reach a low risk value  $\sigma_R \approx 0$  even for  $\rho > -1$ ; in particular, for our choice of correlations  $C$  is a still correlation matrix (i.e. it has a positive spectrum) at  $\rho = -1/N_s = -1/N_w$ . Notice that since  $\partial E_R / \partial \rho = 0$  and  $\partial \sigma_R^2 / \partial \rho = -2 \vec{w}^w \cdot (\vec{\sigma}^w \otimes \vec{\sigma}^s \vec{w}^s) < 0$ , decreasing correlation always decreases the risk of any portfolio. Thus, the efficient frontier moves towards the left as correlations decrease, indicating that for an accurate choice of the correlation patterns the same amount of energy  $E_R$  can be produced with less fluctuations.

### 3.3. Realistic renewable power plants

Finally, we consider a case study based on real solar and wind time data from renewable plants located in Italy (for further information about data the reader can refer to: Mureddu et al., 2015). We consider the 8 sites where both solar plants and wind farms are present. We assume that the standard deviation for wind farms is 15% while for solar plants is 10%; such figures have been suggested by Enel (one of the main electric utility operating in Italy) as realistic values to assess the daily production of renewables (private communication); such values have lead to the agreement of the model of Mureddu et al. (2015) with real data.

We also notice that the total production of wind farms is smaller by a factor  $\sim 7$  respect to the total production of solar plants; hence, portfolio with higher level of energy production will allocate most weights on solar plants.

In Fig. 4 we show the efficient portfolio frontier for the case of



**Fig. 4.** Efficient portfolio boundaries in the case of real solar and wind data from plants located in Italy (Mureddu et al., 2015). The lower curve (black) represents optimal portfolio under the assumption of zero correlation among the risks; the upper curve represent the improvement that can be reached under the same assumption by balancing wind and solar production.

totally uncorrelated energy productions (black curve). It is possible to check that portfolios corresponding to very low productions are wind based, while portfolios with higher energy output are solar based. In the intermediate range, the fluctuations associated to the optimal frontier grow almost linearly with the power output, indicating that intermediate portfolios are mostly a linear combination of low production and high production portfolios.

We then investigate what would balance wind and solar energy production by increasing the size of wind farms, i.e. scaling up their productions and standard deviation by a factor  $\sum e_i^s / \sum e_i^w$ . As seen in Fig. 4, the efficient portfolio frontier for scaled energy sources (red curve) shows that such reallocation allows for a strong reduction of the risk  $\sigma_R$  even in absence of negative correlations. In fact, we are still considering the case of totally uncorrelated energy sources. As can be noticed in Fig. 4, at  $E_R \sim 2900MW$  (corresponding to  $\sim 97\%$  of the maximum possible  $E_R$ ), it is possible to reduce the risk by  $\sim 55\%$  (from  $\sigma_R \sim 300MW$  to  $\sigma_R \sim 170MW$ ).

In general, we can expect that solar and wind power have negative correlations; however, we have checked that if we introduce synthetic negative correlations like in Section 3.2, the efficient frontier remains almost unchanged. This is due to the fact that without reallocation, wind farms are too small and do not produce fluctuations large enough to compensate solar energy's ones. On the other hand, introducing synthetic (anti) correlations in the balanced portfolio has macroscopic effects on the efficient boundary; as an example, for  $\rho = -1/8$  the risk at fixed production  $E_R \sim 2900MW$  of would further reduce by  $\sim 15\%$  (from  $\sigma_R \sim 170MW$  to  $\sigma_R \sim 145MW$ ).

## 4. Discussion and conclusions

In this paper we show that the role of fluctuations when considering the spatial planning of renewable energy sources is not trivial and needs to be properly accounted. By using a basic model based on Optimal Portfolio Theory we show that both with artificially generated data and case studies based on real data, the anti-correlation (often present among the fluctuations of different renewables) can be a suitable criterion for the optimal spacial and temporal allocation of renewable energy production in order to reduce the impact of fluctuations on the size of the electric power balancing market. In fact, thinking in terms of portfolios would allow to optimize also the size and the spatial allocation of future energy accumulation facilities and to allow utilities to develop business models tailored according to the local distribution of renewable sources and storage systems.

Although, in the paper, to highlight the effects of correlations we concentrate on the size of the energy produced as a proxy for the returns – a case where we can rely on the data-set of Mureddu et al. (2015) – it is straightforward to consider the case where power plants have a cost related both to the technology and to the geographical location and where policies attribute a cost to the fluctuations in energy production. In such an approach, portfolios will allow both the policy maker to evaluate the impact on the energy prices and carbon footprint, and investors to correctly estimate their return on the investment.

Furthermore, considering a careful portfolio-driven spatial allocation has several advantages. In particular, an optimal spatial and temporal allocation of renewable energy production could reduce the size of the electric power balancing market with the consequences of lowering average energy prices on balancing markets themselves, since the uncertainties in renewable production would be reduced. Moreover, it would reduce the indirect carbon footprint of renewable sources caused by the use of non-green generators to balance fluctuations in energy production.

Via portfolio analysis it would also be possible to focus on policy issues, like the allocation of non-programmable renewable resources in the most effective locations, helping to optimize the size and spatial allocation of future energy accumulation facilities. With such a framework, it would be possible to minimize the amount of subsidies to renewable generative capacity necessary to reach a given emission reduction goal. Avoiding overcapacity and extending per-unit generation hours would also improve the attractiveness of investment in subsidized renewable generation and provide incentives for the retirement of older and less efficient traditional power generation held for reserve.

A possible extension of the model would be to consider spatial allocations taking into account the energy security of the system also in the case of islanding; in fact, it has been shown that the electric transmission system could take advantage of separating in regional areas in order to mitigate energy congestions and – eventually – black-outs (Mureddu et al., 2016).

In summary, the reduction of fluctuations would lead to several beneficial consequences, like reducing the stress and the congestion on the power grids, maximizing their output by avoiding curtailment, lowering average energy prices on balancing markets, reducing the indirect carbon footprint of renewable sources and optimizing the hours of operations of renewable and conventional energy sources.

Finally, with the inclusion of investment and operative costs of renewable generation and related infrastructures (together with a 5–10 years forecast of trends in electricity demand), the above-mentioned framework could also be adopted as a tool to guide regulators, decision-makers, and utilities in order to attain several goals, like focusing the development of non-programmable renewable resources towards the most effective locations and minimizing the amount of subsidies to renewable generative capacity necessary to reach a given emission reduction goal. Such a portfolio planning, by avoiding overcapacity and extending per-unit generation hours, would also improve the attractiveness of investment in subsidized renewable generation while providing incentives for the decommissioning of older and less efficient traditional power generation held for reserve.

Future directions will be devoted to extending and refining the model for an accurate computation of the correlations testing different distributions for modelling the non Gaussian sources, and extend the computation to the case of a large ( $>10^4$ ) number of micro-generators, exploring a scenario of full deployment of green solar and wind microgrids in urban environments.

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## References

- Bhattacharya, Anindya, Kojima, Satoshi, 2012. Power sector investment risk and renewable energy: a Japanese case study using portfolio risk optimization method. *Energy Policy* 40, 69–80.
- Brouwer, Anne Sjoerd, Van Den Broek, Machteld, Seebregts, Ad, Faaij, André, 2014. Impacts of large-scale intermittent renewable energy sources on electricity systems, and how these can be modeled. *Renew. Sustain. Energy Rev.* 33, 443–466 (May 2014).
- Awerbuch, Shimon, Berger, Martin, 2003. Applying portfolio theory to eu electricity planning and policy making. IAEA/EET Working Paper No. 03, EET.
- Cucchiella, F., Gastaldi, M., Trosini, M., 2017. Investments and cleaner energy production: a portfolio analysis in the Italian electricity market. *J. Clean. Prod.* 142, 121–132.
- DeLlano-Paz, Fernando, Calvo-Silvosa, Anxo, Antelo, Susana Iglesias, Soares, Isabel, 2017. Energy planning and modern portfolio theory: a review. *Renew. Sustain. Energy Rev.* 77, 636–651 (March 2016).
- Facchini, Angelo, 2017. Distributed energy resources: planning for the future. *Nat. Energy* 2 (July), 17129.
- Facchini, A., Rubino, A., Caldarelli, G., Di Liddo, G., in press. Changes to Gate Closure and its impact on wholesale electricity prices: The case of UK. *Energy Policy*.
- Garcia, Reinaldo C., González, Virginia, Contreras, Javier, Custodio, Janiele E.S.C., 2017. Applying modern portfolio theory for a dynamic energy portfolio allocation in electricity markets. *Electr. Power Syst. Res.* 150, 11–23.
- Halu, Arda, Scala, Antonio, Khiyami, Abdulaziz, González, Marta C., 2016. Data-driven modeling of solar-powered urban microgrids. *Sci. Adv.* 2 (1).
- Hammons, T.J., 2008. Integrating renewable energy sources into european grids. *Int. J. Electr. Power Energy Syst.* 30 (8), 462–475.
- Janczura, Katarzyna, 2010. Price volatility and the efficient energy portfolio for the United States. *Atl. Econ. J.* 11293.
- Kennedy, C.A., Stewart, I., Facchini, A., Cersosimo, I., Mele, R., Chen, B., Uda, M., Kansal, A., Chiu, A., Kim, K.-G., Dubeux, C., La Rovere, E.L., Cunha, B., Pincetl, S., Keirstead, J., Barles, S., Pusaka, S., Gunawan, J., Adegbile, M., Nazariha, M., Hoque, S., Marcotullio, P.J., Othar'n, F.G., Genena, T., Ibrahim, N., Farooqui, R., Cervantes, G., Sahin, A.D., 2015. Energy and material flows of megacities. *Proc. Natl. Acad. Sci. USA* 112 (19).
- Kennedy, Chris, Stewart, Iain D., Facchini, Angelo, Mele, Renata, 2017. The role of utilities in developing low carbon, electric megacities. *Energy Policy*.
- Kennedy, Chris, Stewart, Iain D., Westphal, Michael I., Facchini, Angelo, Mele, Renata, 2018. Keeping global climate change within 1.5 °C through net negative electric cities. *Curr. Opin. Environ. Sustain.* 30, 18–25.
- Korjani, S., Mureddu, M., Facchini, A., Damiano, A., 2017. Aging cost optimization for planning and management of energy storage systems. *Energies* 10 (11).
- Krutz, Bert, Van Vuuren, D.P., De Vries, H.J.M., Groeninger, H., 2009. Indicators for energy security. *Energy Policy* 37, 2166–2181.
- Kunz, Friedrich, 2013. Improving congestion management: how to facilitate the integration of renewable generation in germany. *Energy J.* 34 (4), 55–78.
- Markowitz, Harry M., 1959. *Portfolio Selection: Efficient Diversification of Investments*. Yale University Press, New York.
- Mureddu, Mario, Caldarelli, Guido, Chessa, Alessandro, Scala, Antonio, Damiano, Alfonso, 2015. Green power grids: how energy from renewable sources affects networks and markets. *PLoS One* 10 (9), e0135312 (09).
- Mureddu, Mario, Caldarelli, Guido, Damiano, Alfonso, Scala, Antonio, Meyer-Ortmanns, Hildegard, 2016. Islanding the power grid on the transmission level: less connections for more security. *Sci. Rep.* 6 (October) (34797-).
- Mureddu, Mario, Facchini, Angelo, Scala, Antonio, Caldarelli, Guido, Damiano, Alfonso, 2018. A complex network approach for the estimation of the energy demand of electric mobility. *Sci. Rep.* 8 (1) (268-).
- Ortega-Vazquez, Miguel A., Kirschen, Daniel S., 2009. Estimating the spinning reserve requirements in systems with significant wind power generation penetration. *IEEE Trans. Power Syst.* 24 (1), 114–124.
- Pahwa, Sakshi, Scoglio, Caterina, Scala, Antonio, 2014. Abruptness of cascade failures in power grids. *Sci. Rep.* 4 (January).
- Roques, Fabien A., Newbery, David M., Nuttall, William J., 2008. Fuel mix diversification incentives in liberalized electricity markets: a Mean-Variance Portfolio theory approach. *Energy Econ.* 30, 1831–1849.
- Stewart, Iain D., Kennedy, Chris A., Facchini, Angelo, Mele, Renata, 2018. The electric city as a solution to sustainable urban development. *J. Urban Technol.* 25 (1), 3–20.
- Sunderkötter, Malte, Weber, Christoph, 2012. Valuing fuel diversification in power generation capacity planning. *Energy Econ.* 34 (5), 1664–1674.
- Weitemeyer, Stefan, Kleinhans, David, Vogt, Thomas, Agert, Carsten, 2015. Integration of Sources in future power systems: the role of storage. *Renew. Energy*.
- Xu, Bolun, Oudalov, Alexandre, Ulbig, Andreas, Andersson, Goran, Kirschen, Daniel, 2016. Modeling of lithium-ion battery degradation for cell life assessment. *IEEE Trans. Smart Grid*.