

Emergence of anyonic correlations from spin and charge dynamics in one dimensionOleksandr Gamayun¹, Eoin Quinn,² Kemal Bidzhiev,³ and Mikhail B. Zvonarev²¹*London Institute for Mathematical Sciences, Royal Institution, 21 Albermarle Street, London W1S 4BS, England, United Kingdom*²*Université Paris-Saclay, Centre National de la Recherche Scientifique, LPTMS, 91405 Orsay, France*³*PASQAL, 7 rue Léonard de Vinci, 91300 Massy, France*

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We propose a transformation for spin and charge degrees of freedom in one-dimensional lattice systems, constrained to have no doubly occupied sites, that allows direct access to the dynamical correlations of the system. The transformation delivers particle creation and annihilation operators in a form of a spinless particle and a nonlocal operator acting on the space of states of a spin-1/2 chain. This permits a decomposition of dynamical correlation functions as a convolution of those for impenetrable anyons together with those of a spin chain. Further analysis can be done by methods tailored for each part of the convolution, greatly increasing the impact and flexibility of the approach.

DOI: [10.1103/PhysRevA.109.012209](https://doi.org/10.1103/PhysRevA.109.012209)**I. INTRODUCTION**

The physics of many-body quantum systems incorporates effects from interaction and statistics of bare particles. The emerging quasiparticles could inherit the statistics of their noninteracting peers, free fermions turning into a Fermi liquid, and free bosons into a Bose-Einstein condensate. Reducing a system's dimensionality enhances interaction effects and masks out signatures of the statistics of the constituent particles. In one dimension, arbitrarily weak repulsion precludes a macroscopic occupation of a single state with zero momentum, that is, destroys the Bose-Einstein condensate [1]. Furthermore, interactions may transform the bosonic excitation spectrum into a fermionic one, an example being the bosons repelling each other through a δ -function potential of infinite strength, the system known as the Tonks-Girardeau gas, whose excitation spectrum is identical to that of a free Fermi gas [2].

The interplay of spin and charge degrees of freedom could be particularly intricate in one dimension. Systems having a linear excitation spectrum at low energies fall into a Luttinger liquid (LL) universality class regardless of the statistics of the bare particles. Spin and charge degrees of freedom of the microscopic theory are represented by commuting terms in the LL Hamiltonian and factor out in the dynamical correlation functions, the phenomenon referred to as spin-charge separation [3,4]. Accounting for nonlinearities of the excitation spectrum within the effective field theory approach requires proper modification of the LL description, the cases studied recently being spin and charge dynamics above the highly degenerate ground state (spin-incoherent regime [5–7]), in presence of the quadratic branch of the excitation spectrum (itinerant ferromagnetic regime [8–12]), and in the vicinity of the edge of the excitation spectrum [13]. Whether and how the concept of the spin-charge separation may be extended beyond the LL effective field theory description is a challenging open question, relevant, in particular, for ultracold gas experiments [14].

Studying systems with no double occupancy (NDO) constraint (any two particles cannot occupy the same lattice site) is a must for understanding how spin and charge degrees of freedom are coupled at all energy scales. Disregarding the unoccupied sites (“squeezing” the lattice) reduces the space of states of the original system containing N spin-1/2 particles to the space of states of the spin-1/2 chain of length N . The state of individual spins on the squeezed lattice could be controlled and manipulated directly by ultracold quantum gas microscopy [15–17]. On the theory side, some dynamical correlation functions have been evaluated by making use of the coordinate representation for the many-body wave functions, whose structure is very special due to the NDO constraint [18–26]. The formalism of the second quantization, expressing basic microscopic fields of the system in terms of the collective spin and charge variables, could serve as a systemic approach revealing contributions from spin and charge dynamics into any correlation function. However, such a formalism has not been developed so far.

In this paper we present a transformation from the spin-1/2 fermions subjected to the NDO constraint to the collective charge (spinless fermions on a lattice) and spin (spin-1/2 operators on another lattice) variables. These collective charge and spin variables commute with each other, and enter into the transformation in a highly nonlocal way, as shown in Eqs. (9)–(12). Being used for correlation functions, the transformation leads to the charge dynamics of the impenetrable anyons, whose statistical angle is averaged out with the weight function defined by spin configurations.

II. TRANSFORMATION TO SPIN AND CHARGE VARIABLES

We consider spin-1/2 fermions on an infinite one-dimensional lattice. There, $\hat{\psi}_{j\alpha}^\dagger$, $\hat{\psi}_{j\alpha}$, and $\hat{n}_{j\alpha} = \hat{\psi}_{j\alpha}^\dagger \hat{\psi}_{j\alpha}$ are the creation, annihilation, and particle number operators for site j ($-\infty \leq j \leq \infty$), and $\alpha = \uparrow, \downarrow$ is the spin index. The local spin vector $\hat{\mathbf{s}}(j) = (\hat{s}_x(j), \hat{s}_y(j), \hat{s}_z(j))$ can be represented

as

$$\hat{\mathbf{s}}(j) = \frac{1}{2}(\hat{\psi}_{j\uparrow}^\dagger \hat{\psi}_{j\downarrow}^\dagger) \boldsymbol{\sigma} \begin{pmatrix} \hat{\psi}_{j\uparrow} \\ \hat{\psi}_{j\downarrow} \end{pmatrix}, \quad (1)$$

where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the vector composed of the three Pauli matrices. The spin-ladder operators $\hat{\mathbf{s}}_{\pm}(j) = \hat{\mathbf{s}}_x(j) \pm i\hat{\mathbf{s}}_y(j)$ read $\hat{\mathbf{s}}_+(j) = \hat{\psi}_{j\uparrow}^\dagger \hat{\psi}_{j\downarrow}$ and $\hat{\mathbf{s}}_-(j) = \hat{\psi}_{j\downarrow}^\dagger \hat{\psi}_{j\uparrow}$, respectively. We require the total number of fermions in the system, $\hat{N} = \sum_j \hat{n}_j$, to be conserved. Each site can be occupied by either zero fermions or one fermion,

$$\hat{n}_j \equiv \hat{n}_{j\uparrow} + \hat{n}_{j\downarrow} = \{0, 1\}, \quad (2)$$

due to the NDO constraint. The projection operator

$$\hat{\mathcal{X}} = \prod_{j=-\infty}^{\infty} (1 - \hat{n}_{j\uparrow} \hat{n}_{j\downarrow}) \quad (3)$$

eliminates basis states $|\Psi\rangle = \hat{\psi}_{j_1\alpha_1}^\dagger \cdots \hat{\psi}_{j_N\alpha_N}^\dagger |0\rangle$ that contain at least one doubly occupied site. The remaining ones can be uniquely identified as a product of the states $|f\rangle$ and $|\ell\rangle$:

$$|\Psi\rangle = |f\rangle \otimes |\ell\rangle. \quad (4)$$

Here, $|f\rangle = \hat{c}_{j_1}^\dagger \cdots \hat{c}_{j_N}^\dagger |0\rangle$ is defined by spinless fermions on an infinite lattice placed at the positions of the original spin-1/2 fermions. The vacuum $|0\rangle$ for the states $|\Psi\rangle$ and $|f\rangle$ contains no fermions, $\hat{\psi}_j |0\rangle = 0$, and $\hat{c}_j |0\rangle = 0$, respectively. The state $|\ell\rangle = |\alpha_1 \cdots \alpha_N\rangle$ of a spin-1/2 chain of length N can be represented as $|\ell\rangle = \hat{\ell}_-(m_1) \cdots \hat{\ell}_-(m_M) | \uparrow \rangle$. The set $\{m_1, \dots, m_M\}$ indicates the positions of the down spins among $\{\alpha_1, \dots, \alpha_N\}$, M being the total number of the down spins. For example, $|\uparrow\downarrow\uparrow\downarrow\rangle$ gives $\{m_1, m_2, m_3\} = \{2, 4, 5\}$. The vacuum $| \uparrow \rangle$ is the spin-up polarized state. The operator $\hat{\ell}(m) = \boldsymbol{\sigma}(m)/2$ acts on the spin state of the m th particle, and $\hat{\ell}_{\pm} = \hat{\ell}_x \pm i\hat{\ell}_y$.

We now express spin-1/2 fermion fields via operators acting into the spaces formed by $|f\rangle$ and $|\ell\rangle$. The number of particles to the left from the j th site is

$$\hat{\mathcal{N}}_j = \sum_{a=-\infty}^j \hat{n}_a. \quad (5)$$

Here, $\hat{n}_j = \hat{c}_j^\dagger \hat{c}_j$ acting onto $|f\rangle$ corresponds to \hat{n}_j defined by Eq. (2), acting onto $|\Psi\rangle$. Note that the spectrum of the operator $\hat{\mathcal{N}}_j$ is integer valued. Any operator \hat{O} depending on $\hat{\mathcal{N}}_j$ can be understood by the following formula:

$$\hat{O}(\hat{\mathcal{N}}_j) = \sum_{m=-\infty}^{\infty} \hat{O}(m) \delta_{m, \hat{\mathcal{N}}_j}. \quad (6)$$

The operator $\hat{O}(m)$ characterizes the state of the m th particle, and the Kronecker delta

$$\delta_{m, \hat{\mathcal{N}}_j} = \int_0^{2\pi} \frac{d\lambda}{2\pi} e^{i\lambda(\hat{\mathcal{N}}_j - m)} \quad (7)$$

is equal to 1 for the lattice site at which the m th particle is located, and is equal to zero otherwise. The composition law

$$\hat{O}_1(\hat{\mathcal{N}}_j) \hat{O}_2(\hat{\mathcal{N}}_j) = \sum_{m=-\infty}^{\infty} \hat{O}_1(m) \hat{O}_2(m) \delta_{m, \hat{\mathcal{N}}_j} \quad (8)$$

stems directly from Eqs. (6) and (7).

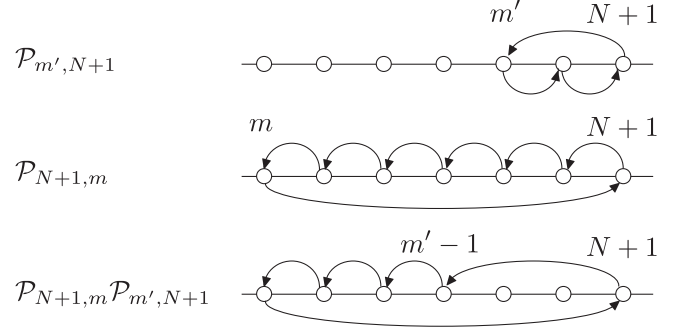


FIG. 1. Shown is the action of the operator \mathcal{P} onto the states of the spin chain. The arrows indicate the directions of the transfer of the local states. The outcome of the action of the composition $\mathcal{P}_{N+1, m} \mathcal{P}_{m', N+1}$ is illustrated for $m' > m$.

We propose the following expressions for the fermion creation operators,

$$\hat{\psi}_{j\uparrow}^\dagger = \mathcal{P}_{\hat{\mathcal{N}}_j, \hat{N}} \hat{c}_j^\dagger, \quad (9)$$

$$\hat{\psi}_{j\downarrow}^\dagger = \mathcal{P}_{\hat{\mathcal{N}}_j, \hat{N}} \hat{\ell}_-(\hat{N}) \hat{c}_j^\dagger, \quad (10)$$

and the corresponding annihilation operators:

$$\hat{\psi}_{j\uparrow} = \hat{c}_j \hat{\eta}(\hat{N}) \mathcal{P}_{\hat{\mathcal{N}}_j, \hat{N}}^\dagger, \quad (11)$$

$$\hat{\psi}_{j\downarrow} = \hat{c}_j \hat{\ell}_+(\hat{N}) \mathcal{P}_{\hat{\mathcal{N}}_j, \hat{N}}^\dagger. \quad (12)$$

The operator $\hat{\eta} = \hat{\ell}_+ \hat{\ell}_- = | \uparrow \rangle \langle \uparrow |$ in Eq. (11) acts on the site of the spin chain defined by the value of the number operator \hat{N} . A way to interpret the dependence on $\hat{\mathcal{N}}_j$ is explained by Eqs. (6) and (7). The cyclic shift operator $\mathcal{P}_{m, m'}$ on a lattice encompassing the sites from m to m' is

$$\mathcal{P}_{m, m'} = \Pi_{m, m+1} \Pi_{m+1, m+2} \cdots \Pi_{m'-1, m'}. \quad (13)$$

The permutation operator $\Pi_{m, m'}$ interchanges the states on the sites m and m' ; in case of spin-1/2 particles it reads

$$\Pi_{m, m'} = \frac{1}{2}[\boldsymbol{\sigma}(m) \otimes \boldsymbol{\sigma}(m') + I \otimes I]. \quad (14)$$

Here, I is the identity matrix. Evidently, Π is its own inverse, $(\Pi_{m, m'})^2 = I$; Hermitian, $\Pi_{m, m'}^\dagger = \Pi_{m, m'}$; and unitary, $\Pi_{m, m'}^\dagger \Pi_{m, m'} = I$. This implies $\mathcal{P}_{m', m} = \mathcal{P}_{m, m'}^{-1} = \mathcal{P}_{m, m'}^\dagger$. The action of the operator (13) onto the states of the spin chain is illustrated in Fig. 1. Note that the local spin operator (1) consists of the pairs $\hat{\psi}_{j\alpha}^\dagger \hat{\psi}_{j\alpha'}$ where $\hat{\psi}^\dagger$ and $\hat{\psi}$ are taken at the same site j . As a consequence, the permutation operator cancels out when using Eqs. (9)–(12), leading to the representation

$$\hat{\mathbf{s}}(j) = \hat{n}_j \hat{\ell}(\hat{\mathcal{N}}_j) \quad (15)$$

already known in the literature [12]. We demonstrate how efficacious are Eqs. (9)–(12) in revealing the contributions from the spin and charge degrees of freedom into the dynamical correlation functions in the remaining part of the paper.

III. HAMILTONIAN

We apply the transformations (9)–(12) to the Hamiltonian

$$\hat{H} = \hat{H}_f + \hat{H}_\ell, \quad (16)$$

where

$$\hat{H}_f = \hat{\mathcal{X}} \left[-t_h \sum_{\substack{j=-\infty \\ \alpha=\uparrow,\downarrow}}^{\infty} (\hat{\psi}_{j\alpha}^\dagger \hat{\psi}_{j+1\alpha} + \text{H.c.}) - h\hat{N} \right. \\ \left. + \frac{1}{2} \sum_{j,j'=-\infty}^{\infty} : \hat{n}_j U_{j-j} \hat{n}_{j'} : \right] \hat{\mathcal{X}} \quad (17)$$

is SU(2) invariant, and the term

$$\hat{H}_\ell = 2B\hat{\mathcal{X}}\hat{S}_z\hat{\mathcal{X}}, \quad \hat{S}_z = \sum_{j=-\infty}^{\infty} \hat{s}_z(j) \quad (18)$$

breaks this symmetry due to the magnetic field B applied along the z projection of the total spin. The symbols H.c. and $:\dots:$ in Eq. (17) stand for the Hermitian conjugate and the normal ordering, respectively. The projection operator $\hat{\mathcal{X}}$, given by Eq. (3), imposes the NDO constraint. Note that the on-site interaction term $:\hat{n}_j^2: U_0/2$ implies an infinite energy cost for having two particles on any site in the $U_0 \rightarrow \infty$ limit. This way, the use of $\hat{\mathcal{X}}$ is equivalent to letting $U_0 \rightarrow \infty$ in the Hamiltonian (16) with no $\hat{\mathcal{X}}$. The actual value of U_0 is irrelevant when $\hat{\mathcal{X}}$ is used, since $\hat{\mathcal{X}} : \hat{n}_j^2 : \hat{\mathcal{X}} = 0$.

Using the transformation (9)–(12) we get Eq. (17) written in terms of the spinless fermions exclusively,

$$\hat{H}_f = -t_h \sum_{j=-\infty}^{\infty} (\hat{c}_j^\dagger \hat{c}_{j+1} + \text{H.c.}) - h\hat{N} \\ + \frac{1}{2} \sum_{j,j'=-\infty}^{\infty} : \hat{n}_j U_{j-j} \hat{n}_{j'} :, \quad (19)$$

and Eq. (18) containing the spinless fermions as well as the spin operators:

$$\hat{H}_\ell = 2B \sum_{j=-\infty}^{\infty} \hat{n}_j \hat{\ell}_z(\mathcal{N}_j). \quad (20)$$

Amazingly, the action of \hat{H}_f (\hat{H}_ℓ) onto the state (4) is nontrivial for the $|f\rangle$ ($|\ell\rangle$) part only:

$$\hat{H}_f |\Psi\rangle = E_f |f\rangle \otimes |\ell\rangle, \quad \hat{H}_\ell |\Psi\rangle = |f\rangle \otimes E_\ell |\ell\rangle. \quad (21)$$

The energy $E_\ell = 2BL_z$, where L_z is the eigenvalue of the operator $\hat{L}_z = \sum_{m=1}^N \hat{\ell}_z(m)$, measuring the z projection of the total spin for the state $|\ell\rangle$ of the spin chain. Hence, the spin degeneracy of the Hamiltonian (16) takes place for any $L_z \neq \pm N/2$. Furthermore, $\hat{H}_\ell = 0$ for $B = 0$, implying 2^N -fold degeneracy as long as the system is not put into a finite volume with some boundary conditions.

IV. FIELD-FIELD CORRELATION FUNCTIONS IN THE THERMAL STATE

We consider the one-body correlation functions, describing the particle propagation,

$$G_p^\alpha(j-j', t) = \frac{1}{Z} \langle \hat{\psi}_{j\alpha}(t) \hat{\psi}_{j'\alpha}^\dagger(0) \rangle_T, \quad \alpha = \uparrow, \downarrow, \quad (22)$$

and the hole propagation,

$$G_h^\alpha(j-j', t) = \frac{1}{Z} \langle \hat{\psi}_{j\alpha}^\dagger(t) \hat{\psi}_{j'\alpha}(0) \rangle_T, \quad \alpha = \uparrow, \downarrow, \quad (23)$$

evaluated at temperature T , chemical potential h , and magnetic field B , on a thermal state:

$$\langle \dots \rangle_T = \sum_{N=0}^{\infty} \sum_{f,\ell} \langle \Psi | e^{-\beta \hat{H}} \dots | \Psi \rangle, \quad (24)$$

where $|\Psi\rangle$ is given by Eq. (4). The sum over f runs through all possible values of the free-particle momenta characterizing the N -fermion state $|f\rangle$. The sum over ℓ runs through all possible configurations of the z projection of the spins, Z is the grand partition function, and $\beta = T^{-1}$. The symmetry

$$G_{p(h)}^\uparrow(j-j', t; h, B) = G_{p(h)}^\downarrow(j-j', t; h, -B) \quad (25)$$

makes it sufficient to evaluate G^\uparrow only.

Using Eqs. (6)–(12) we factorize the matrix element from Eq. (22) into two parts:

$$\langle \Psi | \hat{\psi}_{j\uparrow}(t) \hat{\psi}_{j'\uparrow}^\dagger(0) | \Psi \rangle \\ = \sum_{m,m'=-\infty}^{\infty} \int_0^{2\pi} \frac{d\lambda}{2\pi} \frac{d\lambda'}{2\pi} \\ e^{-i\lambda m + i\lambda' m'} e^{-\beta(E_f + E_\ell)} \mathcal{C}_p(\lambda, \lambda'; j-j'; t) \mathcal{S}(m, m'). \quad (26)$$

The first one encompasses the contributions from the state $|f\rangle$ of spinless fermions:

$$\mathcal{C}_p(\lambda, \lambda'; j-j'; t) = \langle f | \hat{c}_j(t) e^{i\lambda \mathcal{N}_j(t)} e^{-i\lambda' \mathcal{N}_j(0)} \hat{c}_{j'}^\dagger | f \rangle. \quad (27)$$

Its nontrivial time evolution is governed by the Hamiltonian (19). The second one involves the state $|\ell\rangle$ of the spin chain, and the existence of the free fermions is only noticed through their total number N , which defines the length of the chain:

$$\mathcal{S}(m, m') = \langle \ell | \mathcal{P}_{N+1, m} \mathcal{P}_{m', N+1} | \ell \rangle \\ = \langle \ell | \prod_{j=\min\{m, m'\}}^{\max\{m, m'\}} \left[\frac{1}{2} I + \hat{\ell}_z(j) \right] | \ell \rangle. \quad (28)$$

This part is time independent, since the cyclic shift operator, Eq. (13), does not change the value of the z projection of the total spin, L_z . The action of the operator $\mathcal{P}_{N+1, m} \mathcal{P}_{m', N+1}$, illustrated in Fig. 1, leads to vanishing \mathcal{S} if any spin between the sites m and m' is pointed down. This way we get the right-hand side of Eq. (28).

We proceed further by substituting Eq. (28) into Eq. (22) and taking the sum over the spin configurations:

$$\sum_{\ell} e^{-\beta E_\ell} \mathcal{S}(m, m') = \frac{[2 \cosh(\beta B)]^N}{\nu^{|m-m'|}}, \quad (29)$$

where $\nu = 1 + e^{2\beta B}$. We get

$$G_p^\dagger(j - j', t) = \frac{1}{Z} \sum_{\{N\}} e^{-\beta E_f} \int_0^{2\pi} \frac{d\lambda}{2\pi} \frac{d\lambda'}{2\pi} C_p(\lambda, \lambda'; j - j'; t) \sum_{m, m'=-\infty}^{\infty} \frac{e^{-i\lambda m + i\lambda' m'}}{\nu^{|m-m'|}}, \quad (30)$$

where

$$\tilde{E}_f = E_f - \frac{1}{\beta} N \ln[2 \cosh(\beta B)], \quad (31)$$

and the sum over $\{N\}$ encompasses the ones over N and f . The partition function Z can be taken over the fermion configurations f with the energies given by Eq. (31). We have

$$\sum_{m, m'=-\infty}^{\infty} \frac{e^{-i\lambda m + i\lambda' m'}}{\nu^{|m-m'|}} = 2\pi \delta(\lambda - \lambda') F(\lambda; T), \quad (32)$$

where

$$F(\lambda; \nu) = 1 + \sum_{m=1}^{\infty} \nu^{-m} (e^{im\lambda} + e^{-im\lambda}). \quad (33)$$

Therefore,

$$G_p^\dagger(j - j', t) = \int_0^{2\pi} \frac{d\lambda}{2\pi} F(\lambda; \nu) C_p(\lambda; j - j'; t; T), \quad (34)$$

where

$$C_p(\lambda; j - j'; t; T) = \frac{1}{Z} \sum_{\{N\}} e^{-\beta \tilde{E}_f} C_p(\lambda; j - j'; t), \quad (35)$$

and we write $C_p(\lambda)$ in place of $C_p(\lambda, \lambda)$ in order to lighten notations. The summation on the right-hand side of Eq. (35) represents the definition of the thermal state for the spinless fermions with the spectrum given by \tilde{E}_f .

The hole correlation function (23) is treated the same way as the particle one. The result is given by Eqs. (34) and (35) with C_p replaced by

$$C_h(\lambda; j - j'; t) = \langle f | e^{i\lambda \hat{N}_j(t)} \hat{c}_j^\dagger(t) \hat{c}_{j'} e^{-i\lambda \hat{N}_{j'}(0)} | f \rangle. \quad (36)$$

V. EMERGENCE OF IMPENETRABLE ANYONS

The operator $\hat{a}_j = \hat{c}_j e^{-i\lambda \hat{N}_j}$ satisfies the commutation relations

$$\hat{a}_j \hat{a}_{j'}^\dagger + e^{-i\lambda \epsilon(j-j')} \hat{a}_{j'}^\dagger \hat{a}_j = \delta_{jj'}, \quad (37)$$

$$\hat{a}_j \hat{a}_j + e^{i\lambda \epsilon(j-j')} \hat{a}_j \hat{a}_j = 0, \quad (38)$$

where $\epsilon(x) = |x|/x$, and $\epsilon(0) = 0$. This is the fermion-anyon mapping discussed in Ref. [27]. The function $C_p(\lambda)$ turns into

$$C_p(-\lambda; j - j'; t) = \langle f | \hat{a}_j(t) \hat{a}_{j'}^\dagger(0) | f \rangle, \quad (39)$$

which is a correlation function of the impenetrable anyons on a lattice, the variable λ being the statistical angle.

The emergence of the anyon correlation function and its subsequent integration over λ with the function F in Eq. (34)

could be understood as follows. Let us consider a system with M spin-up and $N - M$ spin-down particles. Pick one spin-up particle among them, and pull it through the whole system, subsequently interchanging its coordinate with those of the other particles. The interchanges with the spin-down particles are nontrivial: the spin part of the wave function could give any phase factor since its symmetry is not restricted by the fermion symmetry of the total wave function. We stress that formalizing our *a posteriori* explanation of the structure of Eq. (34) by examining exact finite- N wave functions in the coordinate representations (given, for example, in Refs. [21,28]) goes beyond the scope of the present paper.

Nowadays, there exists a number of proposals to design a system with anyon correlations [29]. They include shaking optical lattices [30], tuning the hopping parameters by the external driving fields [31], and electrical circuit emulation [32]. Our paper demonstrates that anyon correlation can lay behind a transformation of the collective degrees of freedom of a particularly simple interacting system.

VI. PLACE AMONG OTHER APPROACHES

The Hamiltonian (16) with $U_{j-j'} = 0$ represents the exactly solvable $t = 0$ model, also known as the Hubbard model in the limit of infinitely strong repulsion [33]. There, Eq. (34) has been obtained in the form of a Fredholm determinant with the use of the exact wave functions in the coordinate representation [21,28,34]. The transformation (9)–(12) leading to Eq. (34), combined with the ones given in Ref. [35] for the function (27), bring us the same Fredholm determinant representation through much shorter calculations. Note that the model (16) is also exactly solvable when $U_{j-j'} = U \delta_{j, j' \pm 1}$. In this case, the Hamiltonian (19) can be mapped onto the one of the XXZ Heisenberg magnet, and the function (27) can, in principle, be calculated by the Bethe ansatz method.

Special attention has been paid in the literature to the model in the $T \rightarrow 0$ limit, which is highly nontrivial. The ground state of the system is nondegenerate and spin-up (-down) polarized for B negative (positive). In the former case, Eq. (34) describes a spin-up fermion propagating through a gas of the other spin-up fermions. We have $F = 2\pi \delta(\lambda)$ in Eq. (33), hence $G_p^\dagger = \langle \hat{c}_j(t) \hat{c}_{j'}^\dagger \rangle$. In the latter case, Eq. (34) describes a spin-up fermion (an impurity particle) propagating through a gas of spin-down fermions. We have $F = 1$, and the long-time and -distance asymptotic behavior of G_p^\dagger reveals the logarithmic diffusion phenomenon [8,9]. The nondegeneracy of the ground state at $B \neq 0$ stands in sharp contrast to the high degeneracy at $B = 0$, where F is given by Eq. (33) with $\nu = 2$. This regime is known as the spin-incoherent one [5–7]. A challenge put forward in the aforementioned works was to find a low-energy effective field theory, since the low-energy spectrum of spin excitations cannot be linearized for $B > 0$ and $B = 0$, and the LL theory is inapplicable. The representation (34) resolves this problem in the following way: the LL theory is applicable to the function C_p ; the spin excitations are accounted for by the integral over λ with the weight function F without any approximation, which is equivalent to counting the number of worldlines

within the first-quantized path-integral approach implemented in Refs. [6,8].

VII. CONCLUSION

In our paper, we presented a transformation separating collective spin and charge degrees of freedom at a level of fundamental operators entering the Hamiltonian and correlation functions. This approach is exact and applicable regardless of whether the system is integrable or exactly solvable. While the NDO constraint is a must for it to work, the particles constituting the system are not bound to be fermions. In particular, the results for the exactly solvable model of impenetrable bosons [28,34,36] follow readily. As one application, we mention the use of our approach to obtain the correlation function for the investigation of spin diffusion in the one-dimensional Hubbard model [37].

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