# Paying for science: a theory of UKRI grant funding

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How to allocate research funding is an important and contested problem. We analyse the methodology used by UK Research and Innovation to set grant budgets and derive an equation for the unrestricted portion. We find the unrestricted fraction can vary widely, and give a strategy for maximising it. This is important, because the viability of research institutions depends on being able to cover the cost of their research.

Academic research in Britain is funded by UK Research and Innovation. The government agency allocates £8bn to researchers each year, mainly through competitive schemes for research grants. The way in which it does so has several strengths: flexibility in the subject, size and duration of the grant, a simple 12-page application, and moderately high success rates.

UKRI uses a particular methodology for determining grant budgets which, while formulaic, is not intuitive. We analyse this methodology and derive an equation for the portion that is unrestricted. We find that the unrestricted fraction can vary widely, and give a strategy for maximising it. This is important, because the viability of research institutions depends on being able to cover the cost of the research that they do.

Throughout we will use the following terminology:

- $p$ postdoc's annual salary,
- $P$ PI's annual salary,
- $n$ number of postdocs per year on the grant,
- $f$ fraction of time the PI(s) spend on the grant,
- $s$ support costs rate per person-year, set by UKRI.

Note that the number of postdocs per year need not be an integer. For example, if in a three-year grant there are two two-year postdocs, $n = 4/3$. The support costs rate $s$ is independently set by UKRI for different universities and independent research organisations based on their historical accounts. The 2024 lower and upper quartiles for $s$ are £62,809 and £80,431 for non-laboratory research, and £69,815 and £89,872 for laboratory research. In this paper we don't consider the cost of equipment.

## Total and unrestricted income

A typical UKRI grant is between two and four years, but other periods are possible, such as 18 months. However, let's just consider a single year of a grant, which we can linearly scale up to multiple years as needed.

As a simplfying assumption, we take the on-costs of a postdoc and PI to be $1/4$ of the salary. These include the UK National Insurance contribution and any statutory pension, which constitute $1/8$ of the salary, and travel, computing, publication and any immigration fees, which is also $1/8$ of the salary. For example, we assume that a postdoc with a salary of £40,000 per year costs the research organisation $5/4 \cdot £40,000 = £50,000$.

What UKRI calls direct costs are the salary and on-costs of the postdocs and PIs: $5/4\, fP + 5/4\, np$. What UKRI calls indirect costs is the total number of man-years times $s$: $(f + n)s$. UKRI notoriously only funds 80% of the full cost of a research grant. The remaining 20% is meant to be obtained from other sources, such as other grants, student fees or the Research Excellence Framework. So the total amount that is actually applied for—and, if successful, received— is $4/5$ of the sum of the direct and indirect costs, namely,

$$T = (4/5\, s + P)f + (4/5\, s + p)n. \tag{1}$$

The unrestricted portion $U$ of this grant—in the sense of money the research organisation can invest however it wishes, because the PI salaries are sunk costs—is $T$ minus the postdoc costs of $5/4\, np$:

$$U = (4/5\, s + P)f + (4/5\, s - p/4)n. \tag{2}$$

If, for example, we have one postdoc a year ($n = 1$) and a PI spends a quarter of their time on the grant ($f = 1/4$), the equations have a simple and illustrative form: $T = s + P/4 + p$ and $U = s + (P - p)/4$.

Returning to eq. (2), and using the typical values $s = 0.8$, $p = 0.4$ and $P = 0.8$ (all in units of £100,000) gives for the total and the unrestricted portion of the total

$$T = 1.44f + 1.04n,$$
$$U = 1.44f + 0.54n.$$

Here we see the explicit dependence of $T$ and $U$ on the two key quantities $f$ and $n$ that are most under our control. For example, for $f = 1/4$ and $n = 1$, $T = 1.4$ and $U = 0.9$ per year. Over a period of four years, this amounts to a grant of $4T =$ £560,000 with the unrestricted portion being $4U =$ £360,000.

## Maximising unrestricted income

At fixed grant total $T$, we can solve eq. (1) for the number of postdocs per year $n$. Inserting the result into eq. (2) eliminates $n$ in favour of $T$:

$$U = \frac{(4/5\, s - p/4)T + (s + 5/4\, P)pf}{4/5\, s + p}. \tag{3}$$

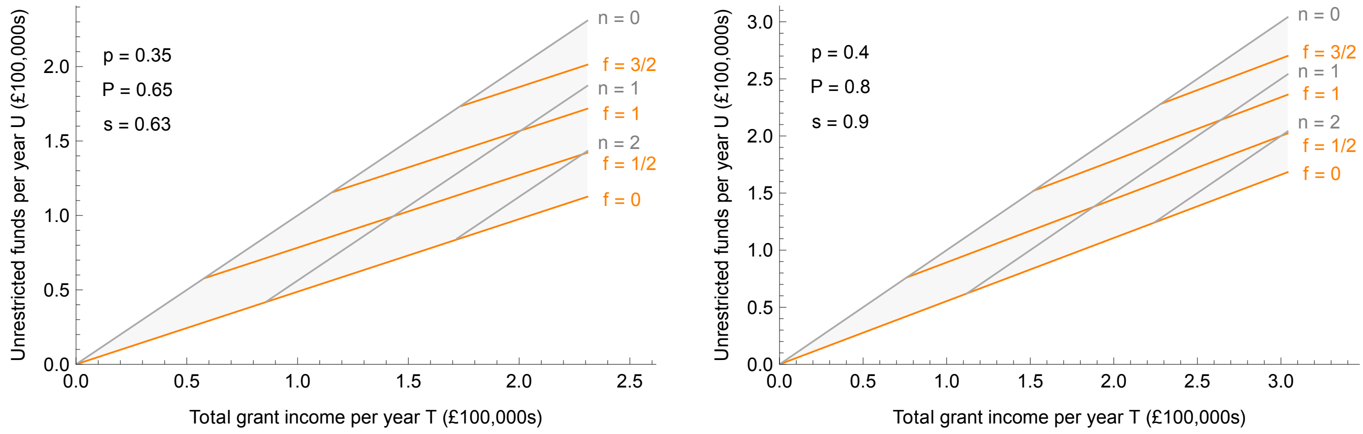


FIG. 1: **Unrestricted funds as a function of total grants funds.** The funds $U$ must lie within the grey cone. The shape of the cone is determined by $p$, $P$ and $s$. Where $U$ lands within the cone is determined by $f$ and $n$. In the left plot we set (in units of £100,000) $p = 0.35$, $P = 0.65$ and $s = 0.63$, and on the right $p = 0.4$, $P = 0.8$ and $s = 0.9$. The orange lines are given by eq. (3) for various values of $f$. The grey lines are given by eq. (4) for various values of $n$.

Plotting this as a function of $T$ gives the orange lines in Fig. 1 for $f = 0$, $1/2$, 1 and $3/2$, with $p$, $P$ and $s$ fixed.

In a similar way, we can solve eq. (1) for the fraction of PI time $f$. Inserting the result into eq. (2) eliminates $f$ in favour of $T$:

$$U = T - {}^4\!/_5\, np. \qquad (4)$$

This form of $U$ we already knew, and indeed we used it to motivate eq. (2). Plotting this as a function of $T$ gives the grey lines in Fig. 1 for $n = 0$, 1 and 2, with $p$ fixed.

The reason that the orange lines cannot pass through the top gray line, and the gray lines cannot pass through the bottom orange line, is because $f$ and $n$ must be non-negative. The touching points are determined by $T \geq ({}^4\!/_5 s + p)n$ and $T \geq ({}^4\!/_5 s + P)f$.

Since $\frac{\partial U}{\partial f}$ in eq. (3) is always positive, to maximise $U$ we want $f$ to be as big as possible. For a single PI, $f \leq 1$, but for multiple PIs, it can in principle be larger. Likewise, since $\frac{\partial U}{\partial n}$ in eq. (4) is always negative, we want $n$ to be as small as possible to maximise $U$.

## Discussion

The main take-home message of this paper is that the amount of unrestricted income $U$ can vary a lot, depending on the choices of $f$ and $n$. It can lie anywhere in the grey cones in Fig. 1, the shape of which is set by $p$, $P$ and $s$. For a given grant total $T$, $U$ increases with $f$ and decreases with $n$.

As an example, consider two versions of a three-year grant, each of which total £528,000. In both versions we set the postdoc salary to be £40,000, the PI salary to be £80,000, and the support costs rate to be £80,000. In units of £100,000, $p = 0.4$, $P = 0.8$ and $s = 0.8$. In the first version of the grant, we choose $f = 1/12$ (so the PI spends a month per year on the grant) and $n = 41/26 = 1.58$ (so there is an average of 1.58 postdocs per year). This gives an unrestricted portion of £291,000, which is 55% of the total. In the second version, we choose $f = 1/2$ and $n = 1$. This gives an unrestricted portion of £378,000, which is 72% of the total.

The fractions above are just two of the values that $U/T$ can take. What is the range of values as we vary $f$ and $n$? The minimum occurs for $f = 0$ and the maximum for $n = 0$, giving the bounds

$$\frac{s - {}^5\!/_{16}\, p}{s + {}^5\!/_4\, p} \leq \frac{U}{T} \leq 1. \qquad (5)$$

For $p = s/2$, this reduces to $27/52 \leq U/T \leq 1$. Multiplying eq. (5) by $T$, the left and right sides are the bottom orange lines and the top grey lines in Fig. 1.

The question of how to balance the amount of PI time $f$ and the number of postdocs $n$ is a matter of personal preference as well as scientific culture. How much should an established scientist invest in research versus training and leveraging young scientists to help out? This depends in part on the nature of the research—experiment, for example, is more labour intensive than theory. It also depends whether we seek an exponential growth in the number of scientists—a movement that began after WWII—or rather seek to keep the number steady. If one in a handful of postdocs goes on to an academic career, then at steady state a PI should have just a handful of postdocs over the course of their career.

The balance also depends on what other duties the PI has. At universities, half of their time might be taken up by teaching, not to mention administrative duties. At the London Institute, where the author works, we do research full-time, so we are inclined to choose higher values of $f$.

[1] https://www.ukri.org/who-we-are/policies-standards-and-data/funding-assurance-programme/.