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Towers of strength

Daniel Rayneau-Kirkhope, **Yong Mao** and **Robert Farr** describe how efficient fractal structures in the natural world are inspiring scientists to develop new materials

The Eiffel Tower was never intended to be a permanent feature of the Parisian landscape. Built for the 1889 World Fair, the plan was to leave the tower in its original location until 1909, when it would be dismantled piece by piece and reassembled elsewhere. However, with the tower still present in 1925, a fraudster called Victor Lustig spotted an opportunity. Posing as a government official, Lustig convinced six scrap-metal dealers that, as the maintenance costs of the tower had grown to outweigh all its benefits, it was due to be scrapped, and he invited them to bid for the job. An auction commenced, but as soon as the "successful" bidder had parted with cash, Lustig fled Paris for Vienna. A month later he returned to repeat the scam, thus becoming notorious as the man who fraudulently sold the Eiffel Tower for scrap metal – twice.

Almost a century after Lustig's con, the Eiffel Tower still dominates the French capital's skyline and is celebrated as one of the world's most iconic structures. However, even if Lustig had been telling the truth, the scrap dealer might still have been disappointed. One of the tower's most remarkable features is its sparing use of material: if all the iron in the 320 m-high Eiffel Tower were melted down to form a solid block with a base area equal to the tower's, the block would stand only 6 cm tall. This surprising fact is a direct result of the tower's architect, Gustave Eiffel, having designed it to be built from iron beams that are much smaller than the tower's finished height. The smallest beams were used

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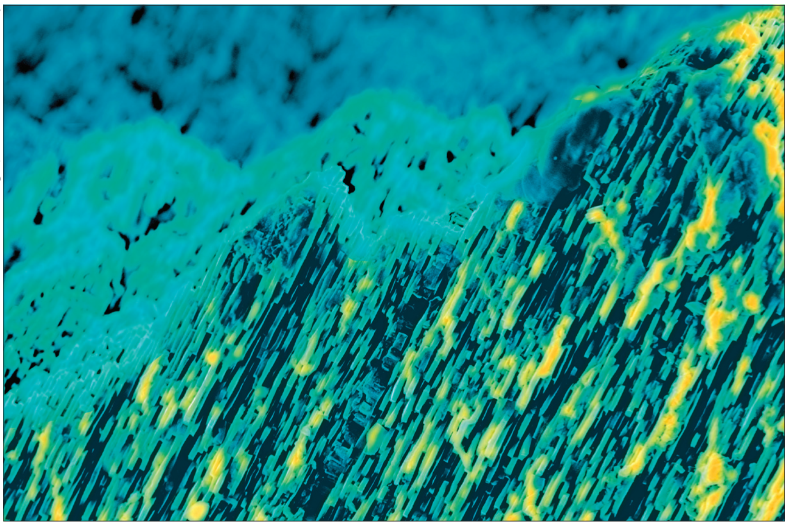

Pearl of great price Mother of pearl, or nacre, is formed from plates of calcium carbonate in a "brickwork" arrangement that enhances the material's toughness and resists cracking.

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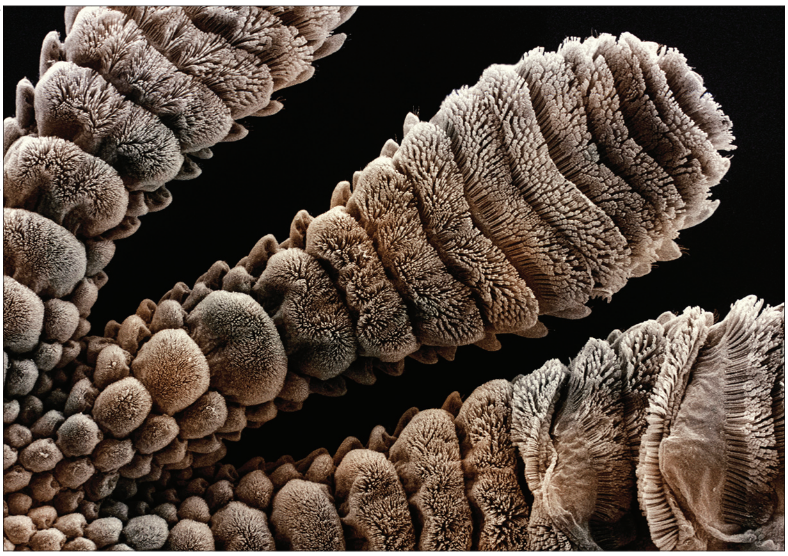

Fuzzy feet This scanning electron micrograph (SEM) shows the ridges and microscopic hairs on the underside of a gecko's foot.

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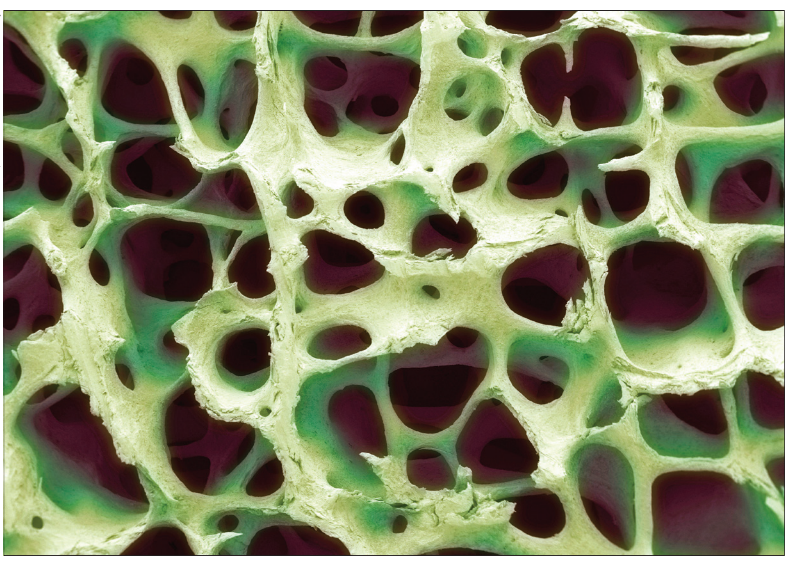

Tough stuff A false-colour SEM of trabecular bone, showing the "honeycomb" structure that gives the bone its strength. The spaces in-between are filled with bone marrow.

to create composite beams, which were then used to create second-order composite structures, and so on for a total of three structural "generations".

This construction method – in which structural elements have a non-trivial internal structure on many length scales – has been termed "hierarchical design", and many of Eiffel's other works also exhibit this same structural hierarchy. The Maria Pia Bridge in Portugal and the Garabit Viaduct in France are perhaps the most striking examples. However, it is not just in architecture that hierarchical design can be advantageous. In other areas of technology and elsewhere, it is often desirable to optimize material properties using particular substructures on multiple length scales. The key questions for designers and scientists are: under what conditions are hierarchical designs beneficial? And what number of hierarchical generations will an optimal structure have?

Natural hierarchies

One strategy for answering these questions is to study examples of hierarchical design in the natural world. The bulk properties of many natural structures show remarkable variation considering the relatively few component materials used in their construction. One example is mother of pearl, or nacre. This naturally occurring composite material is made up of calcium carbonate and a small amount of organic matter in a "bricks-and-mortar" design: hard calcium-carbonate bricks are held together with an organic mortar, producing a composite material with properties far exceeding the sum of its parts. For example, the fracture resistance of nacre is up to 3000 times greater than that of crystals of pure calcium carbonate, thanks to its complex hierarchical arrangement.

Quite a different application for hierarchical design is found on the feet of geckos. These lizards have a remarkable ability to walk up vertical walls and even upside-down on horizontal ceilings thanks to a specific structure found on their toes. The pad of each gecko toe has a series of hairy fibres protruding from it. These fibres repeatedly split into finer and finer "hairs" with dimensions down to the nanometre scale. This repeated splitting increases the surface area considerably, allowing the relatively weak Van der Waals interaction, in conjunction with hydrostatic interactions, to fix the gecko's foot to a surface. Of course, for the gecko to be able to walk, it must also be able to release this adhesive connection. It can do so because the fibre structure is asymmetric: when pulled in one direction, the interaction between the hairs and the surface is strong enough to support the gecko's weight, but when pulled in another, it is greatly reduced.

Some hierarchical structures in nature have an extraordinary property: they appear the same regardless of the level of magnification applied. These structures can be considered fractal over a certain range of length scales. One such structure is found in a particular type of bone known as trabecular, or "spongy", bone. Located throughout the bodies of humans and other animals, trabecular bone is made up of small beams that form an intricate latticework. This complex structure has long been credited as a

1 Stereoscopy

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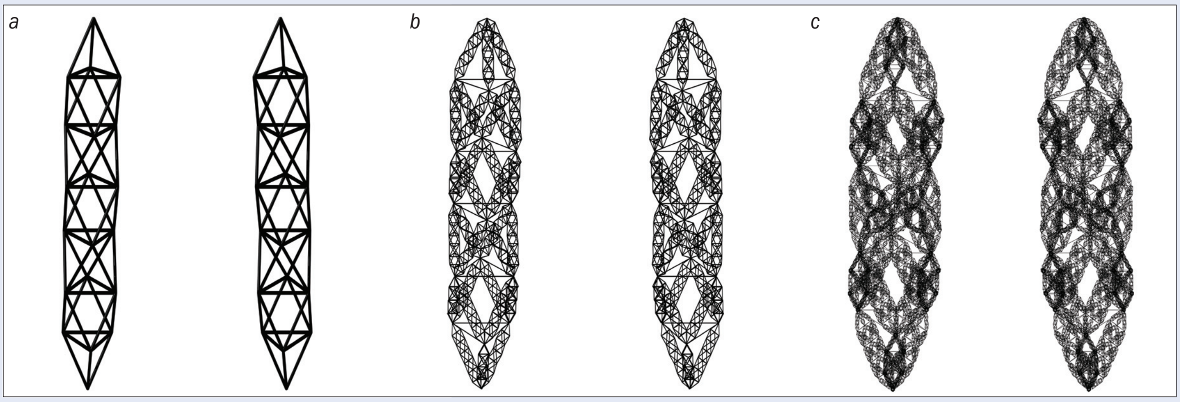

These stereographic images illustrate how different generations of hierarchical frames are created. To view the frames as 3D images, hold the paper around 30 cm from your eyes and focus "through" the page until the images merge. (*a*) A simple generation-1 frame is comprised of *N* octahedra separating two tetrahedra; here, $N = 5$. (*b*) The generation-2 frame is constructed by replacing all compression-bearing beams with scaled generation-1 frames. (*c*) This procedure is repeated for higher generations, such as this generation-3 frame.

source of the bones' strength and low mass, but the exact configuration of the latticework is not specified in full by the genetic code. Instead, the structure is formed in response to prevailing stresses: strenuous use increases the bone mass, while inactivity reduces it, and this optimization routine continues throughout the animal's life. The substructure of the bone has also been found to depend on the magnitude of forces that are applied to it. When the applied loads are relatively small, the bone contains a large number of individual beams, or trabeculae, which take the form of long, slender columns. If the loading increases, the number of trabeculae decreases and they become shorter and stouter.

In order to learn from these remarkable geometries and apply these lessons to our own designs, we must first understand the precise relationship between structure and function in biological materials. For example, a group at the University of Manchester in the UK (building on theoretical work by others) has used a remarkable construction technique to make a reusable adhesive tape based on the gecko's toe. This allowed them to examine directly the effects of density, flexibility, height and fibre diameter (among other parameters) on the structure's adhesive potential. Later, researchers at the University of Akron in Ohio and the Rensselaer Polytechnic Institute in New York used a new fabrication process to create another tape from carbon nanotubes. This tape incorporated an extra level of hierarchy, and was capable of resisting 10 times the shear stress of the original tape – this time outdoing even the gecko's toe.

Design trade-offs

Another, perhaps more rigorous, way of answering questions about optimum structure is to examine the role of hierarchy in specific optimization problems. For example, if we want to support a certain load over a given distance, L, we could ask, "What is the least possible material we require?" To allow more general observations to be made, it is beneficial to define a non-dimensional volume parameter v as the volume of material in the structure divided by L^3. A similar non-dimensional measure of the "heaviness of loading", f, can be made by dividing the force by L^2Y (Y being the Young's modulus of the material). Both parameters are usually smaller than 1 and they are linked by a simple power law relationship, $v \sim f^{\alpha}$, so as f approaches zero, changes in the exponent α have a far greater effect on v than changes to any prefactor.

If the load-bearing structure is under tension, such as a cable supporting a suspension bridge, the amount of material required, v, will be proportional to the loading, f, it must withstand. But for structures under compression, such as columns holding up a building, the situation is a little more complicated. A solid, slender beam with uniform cross-section under compression will deform into a sinusoidal failure mode when the load reaches a critical value; you can see this happen if you stand a plastic 30 cm ruler vertically on its end and press down on it. The fact that the structure can buckle this way means that the volume parameter v no longer scales linearly with the loading factor f. Instead, $v \sim f^{1/2}$ and because f is normally much smaller than 1, more material is required to support a compressive load than a load under tension.

If we replace the solid beam with a hollow tube, keeping the volume of material constant, this sinusoidal deformation can still occur, but only at much higher loading values. However, tubes have their own additional failure mode, known as Koiter buckling, which occurs when there is a local failure of the tube wall – like when an empty beer can is squashed from its ends. After optimizing for tube diameter and thickness, hollow tubes follow a relationship $v \sim f^{2/3}$. Hence, the volume of material required for stability is less than for solid beams, but still greater than that required for structures under tension.

We can increase the power of f (the scaling) still further if we take a hollow beam and – using the principle of hierarchical design – replace it with a "space frame" of hollow beams. The space frame shown in figure 1*a*, for example, is made up of five octahedra with a tetrahedron on each end. It can be described as a "generation-1" structure, meaning that one level of hierarchy has been used to create the structure.

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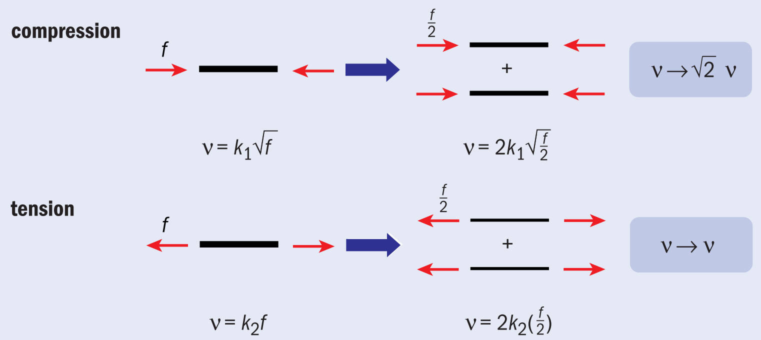
Cost of splitting



When designing a structure, it is often useful to split a load between two or more supporting members. This splitting is often done for structural reasons. For instance, holding up a tent with two or more poles instead of one changes the shape of the tent in a beneficial manner and is worth the increased volume of material; similarly, bridges often have "legs" placed periodically along their span to prevent the collapse of an unsupported region. However, splitting the load in this way can have drawbacks, and the "cost of splitting" for compression- and tension-bearing beams is an important factor in hierarchical design.

To understand this cost, let's look more closely at the relationship between the amount of material in a structure and the loading it can support. The (dimensionless) volume parameter v is related to the loading via a simple power law, $v = kf^{\alpha}$. Here, k is a proportionality constant that depends only on the material's properties and the geometry of the structure; f is a dimensionless loading parameter (and is much less than one for all realistic applications); and α is the scaling of the loading.

For a beam under tension, $\alpha = 1$, so splitting the load does not require increasing the volume of supporting material. In contrast, for a simple, solid beam under compression, $\alpha = 1/2$, so if we use two beams, each supporting half the load over the same distance, the loading parameter for each is halved, but the volume parameter is reduced by only $\sqrt{2}$. Thus, the overall volume of material required increases by $\sqrt{2}$. If we replace the simple beam with a hierarchical structure, the cost of splitting changes yet again. In general, the increase in volume that occurs when two hierarchical beams are used instead of one is $2^{1-(G+2)/(G+3)}$ where G is the generation of the structure.

These different scaling factors have direct consequences for the form of an optimal structure. The most obvious one is that more material is required to support compressive loads than loads under tension. A further consequence is that the higher the number of generations, the less difference it makes as to whether you have one structure holding a given load or two structures holding half the load each.

2 Pressure-bearing structure

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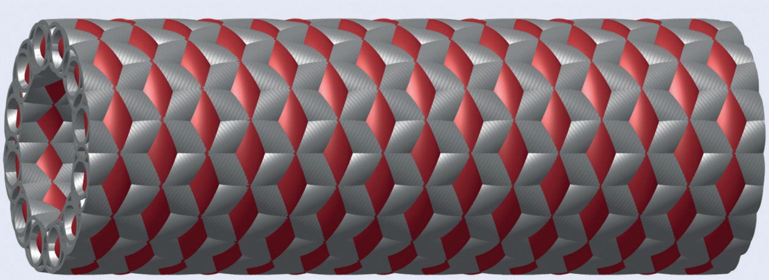


This diagram shows a first-generation pressure-bearing structure. In higher-generation structures, each of the cylindrical sub-structures would be replaced by a scaled version of the whole structure. The design shows remarkable efficiency under gentle pressure loading, and the structure on the largest length scale does not have to be a cylinder – any convex shape constructed with this same substructured material will display the same efficiency gains.

After optimizing the number of octahedra and the radius and wall thickness of the component beams (which are all assumed to be identical), we find that this structure follows a $v \sim f^{3/4}$ power law.

If we continue this procedure of replacing all beams under compressive load with (scaled) space frames constructed from hollow beams, as in figures 1*b* and 1*c*, we find that the scaling is always improved by increasing the level of hierarchy. In general, $v \sim f^{(G+2)/(G+3)}$, where G is the number of generations. Hence, as G tends to infinity, the scaling of volume of material required for a structure to be stable under a given compressive load approaches the best-case scenario of an equivalent tension load.

Does this mean that increasing the generation number is always the way to go? No, because while the scaling increases with generation number, the constant of proportionality between v and f (which depends on the material's properties and the geometry of the structure) does not. The optimal structure is therefore obtained by making trade-offs between scaling and these other factors. Generally, as the loading decreases or the size of the structure increases, the scaling becomes more important and the optimal generation number increases. Conversely, for large loads or small structures, it is sometimes the case that a simple, solid beam is optimal for supporting the load (see box to left).

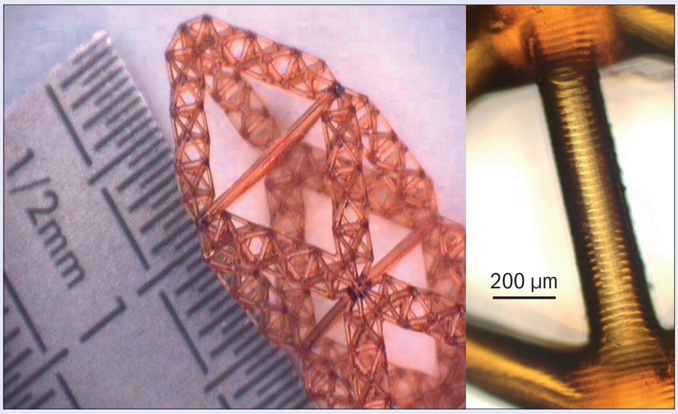
So far we have only looked at structures under compression, but the same patterns emerge when we consider different types of loading. For example, a hierarchical pressure-bearing structure formed from sheets of hierarchically embedded pipes (figure 2) will exhibit the same relationship between generation number and scaling laws, independent of its shape. Consequently, the same trade-offs between scaling and the constant of proportionality must be made to find the optimal generation for the structure.

It is also interesting to look at the patterns that emerge from this optimization process. A structure's fractal dimension is a measure of how details within the structure vary with scale or the efficiency with which it "fills space". Fractal dimension can be measured through a "box-counting" technique in which a 3D space is split into boxes with a characteristic length ε. The minimum number of these boxes required to cover a structure n_ε can then be calculated. The fractal dimension D of a structure can then be defined as $n_\varepsilon \sim \varepsilon^{-D}$, and it is possible to calculate the optimum fractal dimension for a structure under a certain load. In general, it appears that as the load decreases, so does the optimum fractal dimension; in the case of the pressure-bearing structure, the fractal dimension also appears to converge towards a value of two.

Fabricating complex structures

In some ways, this approach merely serves to formalize long-standing knowledge about the relationship between loading and the optimum level of hierarchy. After all, engineers have created chair legs from hollow tubes and cranes out of space frames for a long time, and Eiffel himself used three levels of structural hierarchy in designing his famous tower, long

3 3D printed structure



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This frame was constructed using a 3D printing technique known as stereolithography. In this technique a photopolymer is exposed to light only in regions where solid material is required. The inset shows the layered nature of the material, which is a consequence of the layer-by-layer printing process.

before these ideas had been developed. But this scaling-law approach has helped us to see that the optimum number of generations depends on the loading conditions, and future structures can be designed with this principle in mind.

Promisingly, the technologies required to make these complex designs are progressing apace. Working with Joel Segal of the University of Nottingham, UK, our group at Aalto University in Finland recently used 3D printing technology to produce a generation-2 structure with solid beams (figure 3). The printer uses a photosensitive polymer to build up the structure as a series of individual micron-thick layers, and each beam in it has a radius of a fraction of a millimetre. This structure shows the plausibility of the design and the extent to which modern manufacturing techniques make it possible to design structures with more creative geometries. Another collaboration of experimenters working at HRL Laboratories in California, the University of California and the California Institute of Technology recently created a metallic micro-lattice of hollow beams that was, for a while, the lightest solid in the world. Such experiments can be seen as a first step towards creating metamaterials that would use hierarchical elements in place of hollow tubes, and thus producing structures with unprecedented properties.

Another promising area of research concerns nacre-like materials. It is already possible to fabricate such nacre-mimetics from a wide range of base materials, at least on a small scale, and one of our collaborators' goals is to develop an industrial-scale process for constructing them. By starting with materials that nature does not use for structural purposes, there is a chance that the advantageous properties of nacre and other examples of naturally occurring hierarchical design could even be surpassed by man-made composites. This has already happened with the adhesive tape modelled after a gecko's foot; perhaps the same will be true for nacre, and we will see its mimetics used as a tough, flaw-resistant coating for the aeroplanes of the future. ■