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Economic complexity: Conceptual grounding of a new metrics for global competitiveness

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ABSTRACT

The availability of data corresponding to the products exported by all countries provides an excellent dataset to test economic ideas and extracts new information about the process of economic development. The matrix of countries and exported products shows a marked triangular structure instead of the block-diagonal structure expected from Ricardian arguments of specialization. This observation points to the fact that diversification is instead the dominant effect in the globalized market. We discuss how to define a suitable non-monetary metrics for the value of diversification and the effective complexity of products. We discuss in detail the previous proposed approaches to assess this challenge and their limitations. We introduce a new approach to the definition of these metrics which seems to overcome the previous problems and we test it in a series of model systems.

1. Introduction

A recent strand of empirical literature (Hausmann and Rodrik, 2005; Hidalgo and Hausmann, 2009; Tacchella et al., 2012) which has proposed a *complexity* approach to international trade has emphasized the fact that relatively *rich* countries, i.e. countries ranking high as far as income per-capita is concerned, are also characterized by high diversification of the portfolio of internationally traded goods and services.

As an example of this observation, we report the binary export matrix country–product for the year 2000 in Fig. 1. By binary we mean that we consider a matrix where it is reported whether or not a country is an exporter of a product without considering export volumes. There exist standard methods to consistently define this binary matrix starting from the raw data of export which are usually expressed in US Dollars, defining thresholds over relative measures (Balassa, 1965; Hidalgo and Hausmann, 2009). The source of the raw data on the export flows is BACI's database (Gaulier and Zignago).

According to the basic Ricardian paradigm of international specialization (Ricardo, 1817), countries should export only few products so that the matrix should be block diagonal. This is far from true in reality as shown in Fig. 1.

This scenario resembles biological systems: for organisms and species, there are evidences that diversification usually gives an evolutionary advantage with respect to specialization. Too specialized species tend to become extinct when global

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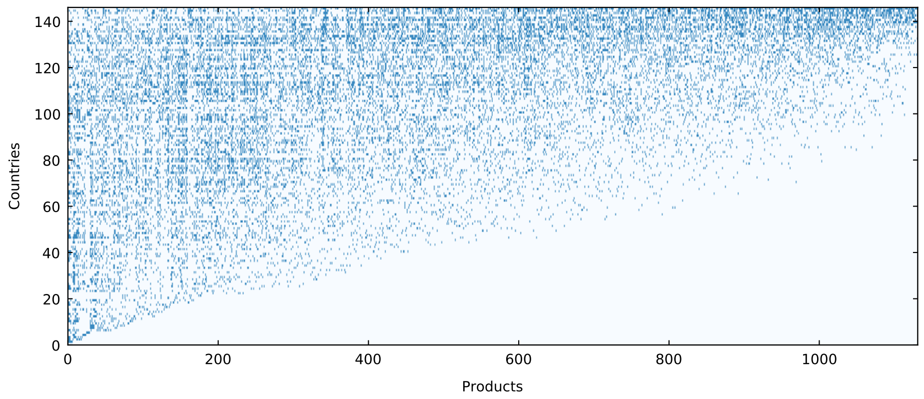

Fig. 1. Binary matrix countries–products: in this matrix we report only whether a country is an exporter of a product or not without considering the export volumes. Basic Ricardian paradigm would predict an almost block-diagonal matrix. The matrix instead has a triangular-like shape. On one hand the most diversified countries result to be the most competitive while the ones specialized on those few products exported by almost every country are the poorest. On the other hand this implies that specialization is an uncommon strategy among most successful countries pointing out that diversification is a more important element to explain the competitiveness and the wealth of countries. The matrix is built from the export flows of the year 1998, the products are categorized according to the Harmonized System 2007 coding system at four digits level of coarse-graining and the digitalization criterion adopted is the revealed comparative advantage (Balassa, 1965).

and abrupt changes occur while species relying on a broader set of resources tend to survive. Similarly for countries, diversification of the productive structure may be due to a new type of competitive advantage – we can label this economic fitness – in the present globalized economic playground. In evolutionary terms, the higher the fitness of a country, the higher the probability of a relatively high income (with respect to countries with lower fitness).

Furthermore the triangular shape gives an additional and specific piece of information about the correlation between the composition of the export basket and the kind of products exported by a country. In fact we find that some countries have a large diversification of their production and consequently make almost all products, while scrolling down the rows of the matrix, the number of exported products decreases and countries become more and more specialized on a small subset of products which are exported by almost all countries.

In such a context, the key issue is therefore the quantitative assessment of the competitive advantage deriving from the diversification of a country. In other words we need to develop a quantitative method to answer the question how many times is the most competitive country more complex with respect to the 10th, to the 40th, to the last, given the matrix countries–products?

A first attempt towards a quantitative definition of the level of complexity of the productive system of a country is presented in Hidalgo and Hausmann (2009) where the authors introduce an iterative method aiming at quantifying the competitiveness of countries and the complexity of products. However, we argue that this method suffers from several conceptual, mathematical and economic flaws. In this paper we focus our attention only on conceptual aspects and discuss the main weak points of the method by the authors of Ref. Hidalgo and Hausmann (2009), called *Method of Reflections*. The economic and mathematical problems deriving from this methods are extensively discussed in Tacchella et al. (2012), Caldarelli et al. (2012) and Cristelli et al. (submitted for publication) where it can be also found the analysis and the results of the new metrics with respect to these two features.

The main reason underlying the failure of the Method of Reflections in correctly measuring the competitiveness of countries from the export matrix M is due to the linear relationship between the country competitiveness and product complexity. Conversely, we argue that a non-linear dependence between these two variables is the fundamental element in order to correctly translate the conceptual framework introduced in Hidalgo and Hausmann (2009) into mathematics. Differently from the Method of Reflections, our metrics defining the country fitness (i.e. competitiveness) and product complexity is the fixed point of two coupled non-linear maps. The non-linear iteration is crucial to bound the complexity of products by the fitness of the less competitive countries exporting them. This non-linear approach is consistent with the triangular shape of the country–product matrix. In this framework the observation that a product is made by a developed country gives a limited information on the complexity of the product itself because these countries export almost all products. On the other hand, when a country with low fitness is able to export a given product, very likely this product requires a low level of complexity. In particular the complexity of a product cannot be defined as the average of the fitnesses of the countries producing it as it happens for the Method of Reflections but must be weighted by the competitiveness of the productive systems of its exporters in a highly non-linear way, so that the information that such a product is produced by some scarcely competitive countries is sufficient to conclude that the complexity of the product is low. Consequently, the only possibility for a product to have a high complexity level is to be produced only by highly competitive countries.

The method represents a new approach for the fundamental assessment of the competitiveness of countries' productive systems and it also introduces a non-monetary measure for product complexity which defines an effective metrics of the value of products filtering out monetary biases such as labor cost, price market speculation, economical inefficiencies of commodities pricing, etc.

The paper is organized in the following way. We briefly discuss the Method of Reflections (Hidalgo and Hausmann, 2009) in Section 2. In Section 3 we discuss the mathematical formulation of the iteration metrics which we propose to be the simplest metrics consistent with the triangular-like pattern of the export matrix and the observation that diversification represents a competitive advantage in a evolutionary system like globalized economies. In Section 4, by three toy models we discuss the conceptual flaws of the method of reflections and why the variables defined in this framework are not a good measure for the competitiveness of countries. In this section we also show that, at least from a conceptual point of view, the metrics correctly grasp the level of competitiveness of countries. In Section 5 conclusions and perspectives are drawn.

2. The Method of Reflections

The Method of Reflections developed in Hidalgo and Hausmann (2009) introduces two variables. The former, k_c, should measure the competitiveness of a country in terms of the diversification of the production while the latter, k_p, is a proxy inversely correlated with the complexity of a product. In such a framework, the more a product is ubiquitous, the higher is the k_p variable and the lower is the corresponding product complexity. From a mathematical point of view the Method of Reflection is defined iteratively as

$$k_c^{(n)} = \frac{1}{k_c^{(0)}} \sum_{p=1}^{N_p} M_{cp} k_p^{(n-1)}$$
$$k_p^{(n)} = \frac{1}{k_p^{(0)}} \sum_{c=1}^{N_c} M_{cp} k_c^{(n-1)} \tag{1}$$

where $k_c^{(0)}$ and $k_p^{(0)}$ are the initial conditions of the iteration and they read as

$$k_c^{(0)} = \sum_{p=1}^{N_p} M_{cp}$$
$$k_p^{(0)} = \sum_{c=1}^{N_c} M_{cp}. \tag{2}$$

N_p and N_c are the number of countries and of products respectively. The matrix M, whose elements are M_{pc}, is built by transforming the raw export flows of US Dollars into unweighted links between countries and products. The criterion adopted in order to understand whether a country can be considered or not a producer of a particular product is the so-called revealed comparative advantage (RCA) that is the fraction of export of the product p by country c with respect to the global export of p done by all countries. This quantity is then divided by the fraction of the total export of c with respect to the whole world export. In order to build the binary matrix M from the RCA matrix, we consider $M_{cp}=1$ if $RCA_{cp} \geq 1$ and zero otherwise. The initial conditions $k_c^{(0)}$ and $k_p^{(0)}$ can be seen as the diversification of the country c (i.e. the number of products exported by a country) and the ubiquity of the product p respectively (i.e. the number of countries which export the product p).

It is worth noticing that in the present discussion and in the following sections we refer to the term *competitiveness* in a evolutionary fashion. In this respect we argue that the export-variety-inspired (i.e. diversification) metrics are promising candidates to address the challenge to quantify the fitness of country productive systems from what they produce.

We point out that Eqs. (1) must be distinguished in *even* and *odd* iterations: the variables $k_c^{(2n+2)}$ of the even iteration can be seen in some way as a refined version (even if they change their meaning) of the $k_c^{(2n)}$. In fact the variables k_c and k_p are alternatively the ubiquity or the diversification of *something*, therefore two iterations are needed to start from a certain type of variable and obtain a variable of the same kind. In the case of $k_c^{(2n)}$ taking only even iterations we are considering only the variables associated to countries which are generalized *diversification*. Equivalently $k_p^{(2n+2)}$ can be interpreted as an ubiquity as $k_p^{(2n)}$ (i.e. as the *dis-value* of the product p). Instead the meaning of the odd iterations is not clear. For a mathematical interpretation of the final state of the even iteration (see Hidalgo and Hausmann, 2009) where it is shown that the Method of Reflections can be formulated in terms of a random walk on the bipartite country–product network. In particular (Cristelli et al., submitted for publication; Caldarelli et al., 2012) the Method of Reflections is equivalent to a trivial eigenvector problem.

In Section 4 (see also Cristelli et al., submitted for publication) we show that the main conceptual flaws of the Method of Reflections are

- variables change their meaning;
- the final state of the variables k_c is not correlated with countries' capabilities;
- the information is shrunk when the method goes towards convergence; and
- given the fact that the fitness of a country is an average, the variable measuring the competitiveness of countries is not extensive with respect to the exported products and therefore the diversification does not necessarily give an advantage to a country.

The first flaw concerns the fact that iterations of an iterative method should refine the information of a given quantity rather than changing, at each time, the definition under investigation. The remaining three instead are due to the linear relationship between product complexity and country fitness in the framework of the Method of Reflections and missing extensively of the country fitness in respect of the number of exported products.

3. A new metrics for countries' competitiveness and products' complexity

In this section we propose a novel iterative method which, on one hand, reflects the original spirit of the theory of capabilities (Hidalgo and Hausmann, 2009) and on the other hand does not suffer from the conceptual and mathematical flaws of the Method of Reflections (Hidalgo and Hausmann, 2009).

In order to formulate a novel metrics for country competitiveness and product complexity, we start from the observation on the relation between diversification of countries and ubiquity of products as shown by the empirical evidence of Fig. 1. We have previously noticed how the triangular structure of the matrix M implies that those countries which export only few products, tend to export those products which are in general exported by almost all countries. These products, in terms of capabilities, should have a low degree of complexity requiring only a small amount of capabilities. On the other hand those countries able to export almost all products are the only ones able to export the most exclusive products. They show in this way to own so many capabilities to be able to produce a large variety of goods from very simple (i.e. low quality/value, requiring few capabilities) to very complex.

This calls for a strongly non-linear relation between the fitness (i.e. competitiveness) and the complexity of the products that they export, leading to the following main argument behind our mathematical approach. While it appears reasonable to measure the competitiveness/fitness of a country through the sum of the complexity of the products of the export basket, the complexity of a product cannot be defined in terms of the average of the fitnesses of the exporters. The definition of the complexity of the product must account for the information that if a poorly diversified country is able to export a given product, very likely this product requires a low level of sophistication. A natural way to achieve such a result is by weighting the complexity of the productive systems of the exporters of a given product through the inverse of their fitness.

The simplest way in which this can be achieved in formulas is the the following way. The iterative method is composed of two steps at each iteration, we first compute the intermediate variables $\tilde{F}_c^{(n)}$ and $\tilde{Q}_p^{(n)}$

$$\tilde{F}_c^{(n)} = \sum_p M_{cp} Q_p^{(n-1)} \tag{3}$$

$$\tilde{Q}_p^{(n)} = \frac{1}{\sum_c M_{cp}(1/F_c^{(n-1)})} \tag{4}$$

and then we define the countries' fitness and the products' complexity at the order n as

$$F_c^{(n)} = \frac{\tilde{F}_c^{(n)}}{\langle \tilde{F}_c^{(n)} \rangle_c} \tag{5}$$

$$Q_p^{(n)} = \frac{\tilde{Q}_p^{(n)}}{\langle \tilde{Q}_p^{(n)} \rangle_p} \tag{6}$$

and the initial conditions are $\tilde{Q}_p^{(0)} = 1 \ \forall p$ and $\tilde{F}_c^{(0)} = 1 \ \forall c$. The variable F is named as fitness and measures the level of competitiveness while the variable Q is the complexity of products. The elements M_{cp} are the elements of the binary country–product matrix M: the element M_{cp} is 1 if the country c exports the product p, 0 otherwise.

The averages at the denominators of Eqs. (5) and (6) are performed on all the *intermediate* values of fitness and complexity defined by Eqs. (3) and (4) respectively. At each iteration both variables (fitness and complexity) are renormalized to keep constant the total export (that is, the average export per country as the number of countries is fixed) and the average complexity of products respectively. We introduce this renormalization procedure because our non-linear equations have two trivial absorbing solutions in 0 and $+\infty$. By the renormalization of the space in which the values of fitness and complexity lie we are able to grasp the non-trivial, self-consistent and economically meaningful solution.

Firstly we note that the iterations do not change the meaning of the variables, each iteration only refines the information. As a second point, in the computation of the complexity of a product, the weight assigned to countries is inversely proportional to the fitness, in such a way we properly take into account the philosophy of capabilities. The countries with small fitness dominate the sum in Eq. (4) as expected. Thirdly, the diversification gives an advantage in terms of country fitness being the F_c extensive with respect to the number of products exported. As a final remark this version of our method

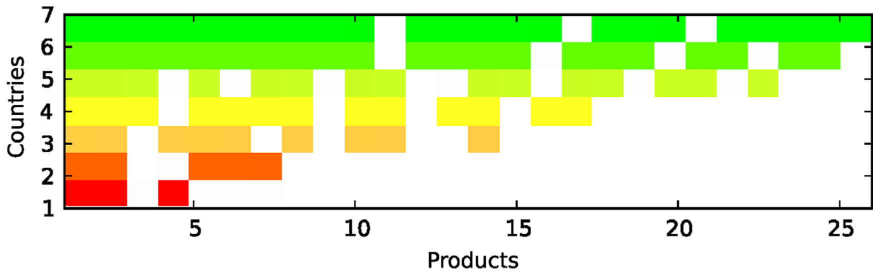

Fig. 2. The matrix countries–products: we build a triangular-shaped matrix with seven countries and 26 products in order to study the k_c and F_c variables as a function of the order of the iterative method, the Method of Reflections and the method discussed in this paper respectively. The colors of rows and their intensity are assigned according to the diversification, the same color scheme is adopted in Fig. 3 in order to follow the evolution the variables of both methods corresponding to the same country.

can be seen as the intensive case since the matrix M is a binary matrix which does not take into account the amount of export of a country. Thus the metrics deriving from the binary matrix is purely non-monetary, this metrics intensively measures the productive power of countries.

4. Case study

In this section we discuss three simple toy models in order to compare the metrics discussed in this paper for the country complexity with the one in Hidalgo and Hausmann (2009). In fact we aim at testing which metrics is consistent with the conceptual framework of the capabilities. We propose three simple tests in order to:

- discuss how the variables characterizing the complexity of countries converge to the self-consistent solution for both methods;
- test whether or not the k_c variables and the fitnesses F_c get correlated with the endowment of capabilities of countries; and
- compare the complexity of products in both methods in a small world composed of only four countries.

4.1. A synthetic world with 7 countries and 26 products

Let us consider a synthetic economic system composed of seven countries and 26 products. These countries and products do not correspond to any real data, they are a toy world economic system mimicking the features of the real one shown in Fig. 1. As for the real export data, we can characterize this system by introducing a binary 7×26 M matrix in which the element M_{cp} is, as usual, 1 if the country c exports the product p, 0 otherwise. We define the M matrix of this synthetic system in order to mimic the triangular feature of the real M matrix (see Fig. 2 for a graphical representation of the 7×26 matrix used, colors and their intensity are assigned according to the diversification, green to the highest diversification, red to the lowest one). Therefore the most competitive countries of our system have a large diversification of their production and consequently they make almost all products while decreasing the competitiveness of these synthetic countries, the production becomes more and more specialized on a small subset of low complexity products which are exported by almost all countries.

This very simple model allows us to illustrate the action of the iteration defined in this paper in Eqs. (5) and (6) with respect to the ones introduced in Hidalgo and Hausmann (2009). Given the triangular structure of the M matrix, one should expect that highly diversified countries export products whose average complexity is higher than the ones exported by countries specialized on those few products exported by almost all countries. Consequently one also expects that the iterations of a good candidate to be the metrics for capabilities should tend to enhance the distance between countries, especially if they are very far from a diversification point of view.

In Fig. 3 (right panel) we show the results of the iterations defined in Eqs. (5) and (6) applied to the synthetic 7×26 binary matrix M and the ones produced by the iterations (left panel) as defined in Hidalgo and Hausmann (2009). The color scheme is the same of Fig. 2. While the linear iteration defined by the authors of Hidalgo and Hausmann (2009) unavoidably drives all the fitnesses of the seven countries (see Caldarelli et al., 2012; Cristelli et al., submitted for publication for further mathematical considerations about this convergence) to the same value, our method instead enhances, as expected, at each step the differences among countries, supporting the idea that each iteration refines the information and tends to magnify the distance of countries once the information about the product complexity is taken into account. A similar scenario is observed for product complexity too. The distribution of the product complexity is shrunk by the method in Hidalgo and Hausmann (2009) towards a delta-function while our method tends to broaden the distribution.

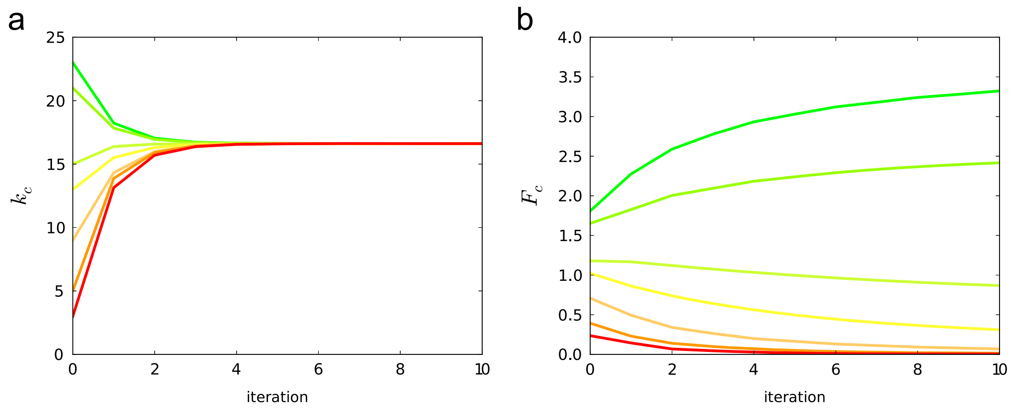

Fig. 3. Left panel: the distribution of the k_c variables is shrunk by the methods in Hidalgo and Hausmann (2009) towards a delta-function, a similar behavior is observed for k_p. Only even iterations are reported. *Right panel*: our method defined by Eqs. (5) and (6) instead enhances, as expected, at each step the differences among countries. Each iteration refines the information and tends to magnify the distance of countries once the information about the product complexity is taken into account. In both panels the color scheme is the same of Fig. 2. Therefore looking at the same color in both panels we are able to follow the evolution of the variables corresponding to the same country.

It can be shown that the two metrics (i.e. F_c and Q_p) are not asymptotically diverging and represent the fixed point of the non-linear coupled maps defined in Eq. (5) whose stability with respect to the initial conditions has been tested numerically.

The shrinkage of the information of the Method of Reflections can be traced back to the fact the iterative procedure introduced by Hidalgo and Hausmann can be formulated in terms of an eigenvector problem. In this eigenvector problem the asymptotic values of k_c and k_p are proportional to the components of the eigenvector associated to second largest eigenvalues. Given a series of mathematical considerations, it can be shown that only the largest eigenvalue has a real part equal to one while all the remaining ones have a real part less than one. Therefore the shrinkage of the Methods of Reflections is due to the equivalence of k_c and k_p variables with a vanishing component of the eigenvector problem.

As said, all the phenomenologically defined quantities $k_c^{(2n)}$ run to a constant value independent of the country index c at increasing order $2n$. The same happens for $k_p^{(2n)}$. This problematic behavior motivated the authors to correlate at a given order $2n$ not the entire quantity $k_c^{(2n)}$, but its fluctuation from the arithmetic mean over all countries, normalized by the standard deviation of the same quantity. This operation accounts for the subtraction of the fixed point of the iterations, while the second one eliminates practically the contraction coefficient of the fluctuation. In our opinion it is rather odd that in an empirically defined algorithm involving phenomenological quantities the observables of the phenomenon are the vanishing fraction of them.

4.2. Capabilities: testing the metrics

In this section we show how the fitness F_c of a country as introduced in this work correlates with the capabilities. In fact in the spirit of Hidalgo and Hausmann (2009) the aim would be to obtain self-consistent metrics (in Hidalgo and Hausmann, 2009 the k_c variables, in this work the country fitness F_c) able to grasp and measure the level of complexity of a productive system. Unfortunately in the case of real country–product matrices we cannot directly access to the vector of capabilities owned by a country. However, by a toy model we can construct a synthetic world in which we assign the capabilities owned by a country and the capabilities required by a product in order to measure the degree of correlation between the endowment of capabilities and the fitness or the k_c variables.

In this respect we define two matrices: a country-capability matrix, whose entries C_{ca} specify which capabilities are owned by a country and a capability-product matrix, whose elements P_{ap} specify which capabilities are required to make a product. Performing the matrix product $C \times P$ of these two matrices and calling T the result of this product, we define the binary country–product matrix M whose entries have the same meaning of the a real matrix M in the following way:

$$M_{cp} = \begin{cases} 1 & \text{if } T_{cp} = \sum_a P_{ap} \\ 0 & \text{if otherwise.} \end{cases} \tag{7}$$

Differently from the matrix extracted from real export flows now we know the number of capabilities n_c of each country by construction ($n_c = \sum_a C_{ca}$). Then we apply the iteration method here introduced (see Eqs. (5) and (6)) to this binary matrix and compute the correlation among the fitnesses F_c of a country and the respective number of capabilities n_c.

The C and P matrices are defined as in Hidalgo and Hausmann (2009). Following Hidalgo and Hausmann (2009) the matrices C and P are random matrices. The elements of C are 1 with probability r=0.76 and 0 with probability $1-r$. Instead the elements of P are equal to 1 with probability q=0.035 and 0 with probability $1-q$. In this synthetic system we consider

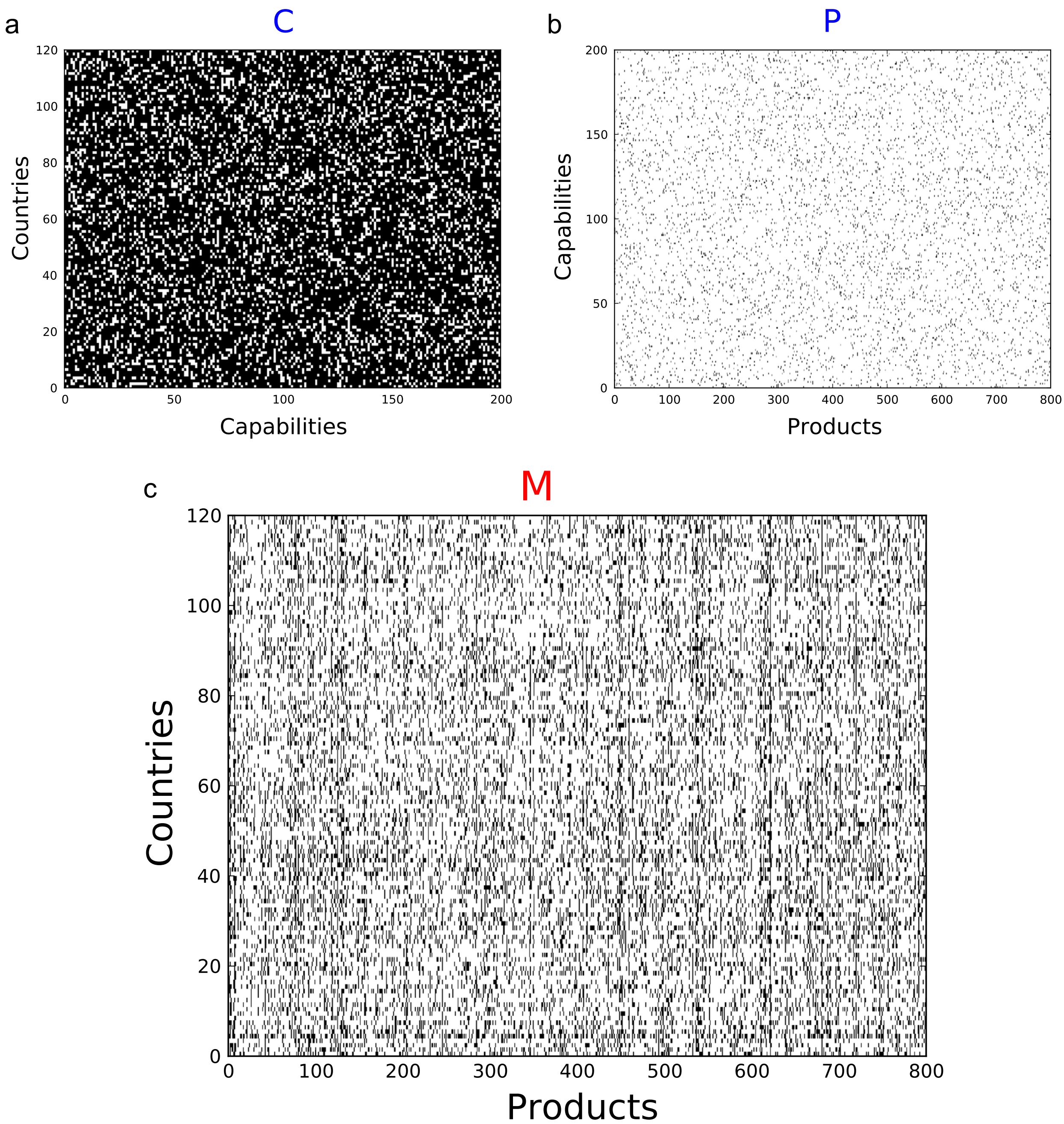

Fig. 4. Graphical illustration of the matrices C, P and M: the matrices C and P are binary random matrices which define the capabilities owned by a country and the capabilities required by a product respectively. The resulting M matrix is used to test the correlation among the capabilities owned by a country and the corresponding complexity measured by F_c and k_c.

200 capabilities while the number of countries and products is equal to 120 and 800 respectively. In Fig. 4 we give a visual representation of the three matrices P, C and M.

In Fig. 5 we plot the correlation between capabilities and fitnesses (green solid line) and k_c (red solid line) as a function of the iteration order. As discussed in Hidalgo and Hausmann (2009), Tacchella et al. (2012) in the figure we report only the even iterations of the variable k_c because only even iterations are positively correlated with the complexity of a country.

On one hand the k_c variables get uncorrelated with the number of capabilities owned by a country increasing the number of iteration. According to this toy model the k_c variables are not able to measure the level of complexity of a country and the Method of Reflections destroys the initial correlation.

Conversely the metrics introduced in this paper shows an almost constant correlation so that the iterative equations here introduced permit to catch at least the same information of the starting point . In this specific example the method does not

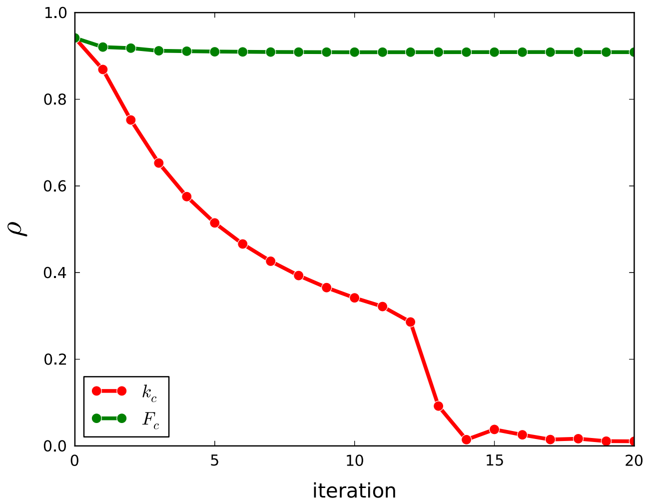

Fig. 5. Correlation between the number of capabilities of a country n_c and the corresponding complexity measured by k_c and F_c respectively: increasing the iteration order, the Method of reflections destroys the initial correlation between the initial state (i.e. the diversification) and the number of capabilities owned by countries (red line). Instead the correlation between F_c and capabilities is constant (green line). Our method does not provide any additional information (i.e. growth of the correlation) because the matrix M is defined by two random matrices therefore it does not contain any piece of information. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

Table 1
The fitnesses of the countries of the small world.

Country	Fitness
USA	4
Austria	3
Mexico	0.5
Kazakhstan	0.1

Table 2
Resulting complexity of products: k_p vs. Q_p.

Producers	k_p	Q_p
USA, Austria	3.50	1.71
USA, Mexico	2.25	0.44
Austria, Mexico	1.75	0.43
USA, Austria, Mexico	2.5	0.39
USA, Kazakhstan	2.05	0.098
Austria, Kazakhstan	1.55	0.097
…	…	…
USA, Austria, Mexico, Kazakhstan	1.90	0.080

provide any additional information because the matrix M is defined starting from two random matrices therefore it does not contain any piece of information.

4.3. *Hierarchical complexity of products*

We consider a synthetic world made up by only four countries: USA, Austria, Mexico and Kazakhstan. We assume that each of them has a fixed fitness F_c as reported in Table 1.

We then perform a single step of the Method of Reflections, thus evaluating $k_p = \langle F \rangle$, and a single step of our method: $Q_p = \sum_c (1/F_c)^{-1}$. The average and the sums are performed only over the countries producing the pth product. If we calculate Q_p and k_p for products made by every possible combination of the four countries, the results obtained are reported in Table 2 (Q_p and k_p are both equal to F when the producer is only one).

We clearly see that the ranking defined by Q_p is strongly dependent on who is the worst of the producers, which seems reasonable if we believe that the complexity of the product is strictly linked with the smallest set of capabilities needed to produce it. The k_p variable instead does not show this *hierarchical* behavior being simply defined by the average of the fitness of the producers. We also note that within the clusters of values related with the worst producers, an internal ordering is present, which depends on the ubiquity of the products and on the fitness of the other producers, for instance see the highlighted cells in Table 2. Instead the variable k_p completely misses this aspect predicting the highest complexity of the product with the highest ubiquity (USA, Austria, Mexico) among all the possible combinations of exporters having Mexico as the worst country.

5. Conclusions

In this paper we have discussed a novel method to define a self-consistent and non-monetary metrics for the competitiveness of countries and the complexity of products. Differently from the Method of Reflections (Hidalgo and Hausmann, 2009), this new metrics is able to grasp the level of competitiveness of a country. In order to consistently translate the conceptual framework of economic complexity we use a non-linear algorithm whose fixed point is the metrics for countries' fitness and products' complexity. The non-linear coupling between fitness and complexity is crucial to correctly assess the consideration that the simple fact that a product is made by a developed country does not convey significant information on the complexity of the product, while the information that a poor country is able to export a product means that this product requires a low level of sophistication and few capabilities. In practice in the definition of the complexity of the product we weight the exporters of this product with the inverse of their fitness.

The variables of the Method of Reflections are compared with the new metrics analyzed in this paper and proposed for the first time in Tacchella et al. (2012). The main results are:

- the distributions of the k_c and k_p are shrunk by the Method of Reflections and asymptotically driven towards a delta-function. Eqs. (5) and (6) instead tend to broaden the distribution of F_c and Q_p with respect to the initial state towards Pareto-like distribution;
- by defining a synthetic tripartite network and assigning the number of capabilities owned by a country we observe that the Method of Reflections destroys the correlation between the capabilities and the variables k_c increasing the order of the iteration, differently from Eqs. (5) and (6);
- the k_p variables are not able to correctly measure and reflect the complexity of products differently from the Q_p variables as shown by a toy model in which only four countries are considered; and
- our approach instead seems to correctly measure the level of capabilities.

The application of this method to real country–product matrices can provide important and new information on one hand about economic systems and competition among countries and on the other hand may be used as a tool for fundamental analysis of financial markets. These applications are and will be discussed in other works (see Caldarelli et al., 2012; Tacchella et al., 2012; Cristelli et al., submitted for publication).

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